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DESAP 1 - A STRUCTURAL DESIGN PROGRAM WITH STRESS AND DISPLACEMENT CONSTRAINTS

Volume II: Sample Problems

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16. ABSTRACT

DESAP 1 is a finite element program for computer-automated, minimum weight design of elastic structures with constraints on stresses (including local instability criteria) and displacements. Volume 1 of the report contains the theoretical and user's manual of the program. Sample problems and the listing of the program are included in Volumes 2 and 3, respectively.

The static analysis portion of DESAP 1 is based on the SOLID SAP finite element program developed at the University of California, Berkeley. In design, the stress ratio method is employed for the stress constraints, whereas the displacement constraints are handled by solving the appropriate optimality criterion.

The element subroutines have been organized so as to facilitate additions and changes by the user. As a result, a relatively minor programming effort would be required to make DESAP 1 into a special-purpose program to handle the user's specific design requirements and failure criteria.

DESAP 1 is a companion program of DESAP 2, "A Structural Design Program with Stress and Buckling Constraints." With the exception of a few cards, the same data deck can be used for both programs.

This is Volume 2 of three volumes.

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K. PREAMBLE

This collection of sample problems is a supplement to Volume 1 of DESAP 1: "Theoretical and User's Manual". In making up this volume, our aim was to find examples that would best serve the following functions:

1) Illustrate and supplement the input instructions of Volume 1, and to familiarize the user with the output.

2) Explain, with examples, special problem areas and peculiarities that may arise in the use of the program.

3) Provide example problems that may be used for debugging the program during installation on a new computer system.

4) Compare the results of DESAP 1 against solutions obtained by other means, whenever possible.

Although DESAP 1 is designed primarily for the use of large structures, the stated purpose of the sample problems is clearly best fulfilled by small, simple examples that do not necessarily represent realistic design situations. Consequently, the problems appearing in this volume should be viewed strictly as tools of instruction, which in no way reflect the ultimate capabilities of the program.

Because our experience with the program is rather limited at this time, the example problems may well have overlooked some troublesome aspects of design, or even deficiencies in the program itself. The extensive computer output from each design cycle is, however, a powerful diagnostic tool that should enable the user to pinpoint the difficulty and make the appropriate correction.

An example problem is given for each element type presently used in the program. Each problem contains a complete description of the input data, including an echo of the input cards, and the computer printout of the input information. In order to reduce the bulk of the report, only a partial listing of the computer output is duplicated, containing the initial and the final designs. The complete history of a design is usually summarized by tabulating the design variables.

In compiling the sample problems, we were seriously handicapped by a lack of adequately documented optimal design problems in existing literature. For this reason, a one-to-one comparison of the results of DESAP 1 with independently obtained solutions is lacking in many of the problems.

As a final note, we would like to remind the user again that DESAP 1 is oriented towards large problems. Mainly due to an extensive use of auxiliary storage devices and other core-saving features, the program is not efficient for small structures as used for the sample problems. Consequently, the computer times for these problems are not expected to be competitive with runs obtained from programs especially designed for structures of limited size.

L. BAR ELEMENTS

L.1 Ten-Bar Cantilever Truss

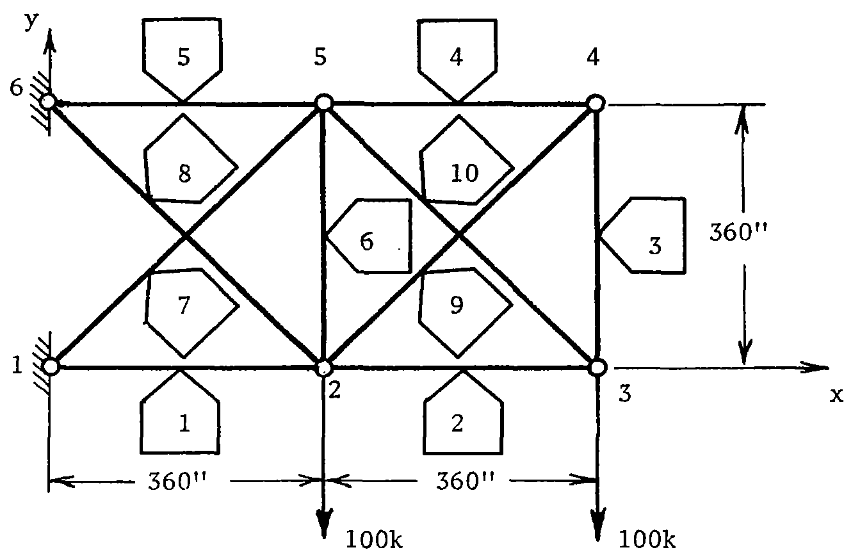

Figure L.1.1

Layout of Cantilever Truss

The truss shown in Fig. L.1.1 is subjected to a single load condition also defined in the figure. All the members are to be sized independently. The data employed in the design is:

$E = 10^7$ psi (Young's modulus).

$\sigma_t^* = \sigma_c^* = 25{,}000$ psi (allowable stress).

$\rho = 0.1$ lb/in^3 (specific weight).

$A^* = 0.1$ in^2 for all members (min. allowable cross-sectional area).

$A = 20$ in^2 for all members (initial cross-sectional area).

$u_y^* = \pm 2$ in. for nodes 2, 3, 4 and 5 (max allowable displacements in $\pm$ y-directions).

Local buckling of members is not considered as a design criterion.

It is known from previous treatment of the problem [9,10] that the design converges slowly---24 design iterations were reported in [9]. The slowness is caused by the presence of passive members (governed by the minimum size constraints) in the final design, combined with unrealistically small minimum allowable sizes.

In view of this prior knowledge, it was decided to over-relax the displacement-constrained design, thereby reducing the number of design cycles. The design was carried out in two stages. In the first stage $\alpha = 0$ was used for one cycle (NCYCL = 1), and a restart deck was requested (KPUNCH = 1). The restart deck was then employed to start the second stage with $\alpha = 0.25$; the minimum weight design was obtained after five additional redesigns. The history of the design process has been summarized in Fig. L.1.2 and Table L.1.1.

Special notes on input-output:

1) Uniform scaling is an exact operation for this particular problem, since all the element stiffness matrices have the form $[K_i] = [k_i]A_i$. Consequently, KSCALE = 1 was used in Design Control Data.
2) In the absence of local buckling in the stress-constrained design, the distinction between Construction Codes Nos. 1 and 2 vanishes. We chose quite arbitrarily KODE = 1.
3) Local buckling of members was eliminated as a design consideration by leaving the moments of inertia blank on the Geometric Property Cards. The blanks were replaced by the computer with $I_{\bar{y}} = I_{\bar{z}} = 10^6$, which in turn results in very high buckling strength.

4) A negative displacement ratio (see Evaluation of Design No. 0) indicates that the ratio is determined by a displacement constraint in the negative coordinate direction.

5) Load Condition = 0 (see Evaluation of Design No. 5) denotes that the stress ratio is determined by a minimum size constraint.

6) The design procedure was terminated when a weight increase was detected between Design Nos. 5 and 6 (see Evaluation of Design No. 6). This increase is due to the appearance of an active stress constraint which tends to push the design past the minimum weight point towards the fully stressed design.

Element	Critical, Scaled Designs (sq. in.)							
	Stage 1 (α=0)		Stage 2 (α=0.25)					
	0	1	1	2	3	4	5	6
1	20.0	54.65	27.69	27.29	24.80	23.86	23.79	27.81
2	20.0	5.91	16.88	11.80	15.23	14.41	14.96	18.15
3	20.0	3.21	5.09	2.82	1.82	0.63	0.32	0.16
4	20.0	3.21	5.09	2.82	1.82	0.73	0.10	0.12
5	20.0	52.60	27.15	30.68	29.85	31.11	30.58	37.41
6	20.0	2.57	0.55	0.13	0.10	0.10	0.10	0.12
7	20.0	17.26	15.70	19.76	19.63	21.44	20.83	26.05
8	20.0	18.48	14.73	13.31	9.88	8.14	8.46	7.92
9	20.0	6.42	6.16	4.09	2.56	1.03	0.12	0.12
10	20.0	11.83	18.87	18.23	20.46	20.91	20.88	25.84

Table L.1.1

Design History of Cross-Sectional Areas.

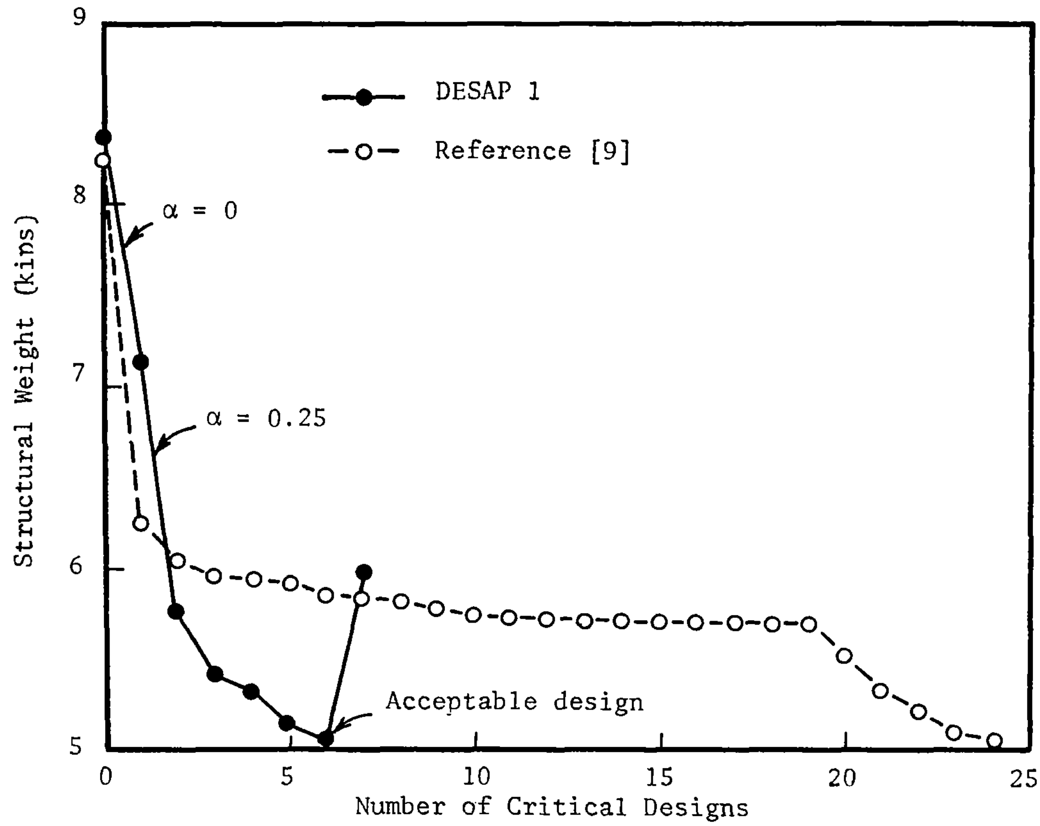

Figure L.1.2

Cantilever Truss Design History of Structural Weight.

```
00001 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
16900  STRESS & DISP. CONSTRAINTS - 10 BAR CANTILEVER  - VENKAYYA,S EX.4,CASE. 2
16950     6    1    1   10
17000     1         1 0.01       0.01        1    1    1 0.8
17050    10  20.0      0.1
17100     1    1    1   -1   -1   -1   -1
17150     2                                360.
17200     3                              720.
17250     4                              720.       360.
17300     5                               360.      360.
17350     6    1    1    1    1    1    1           360.
17400     1   10    1    1    1    1
17450     1    1 0.1
17500            10000000.           25000.    -25000.
17550     1       1.
17600
17650
17700
17750
17800     1    1    2    1    1    1
17850     2    2    3    1    1    2
17900     3    3    4    1    1    3
17950     4    4    5    1    1    4
18000     5    5    6    1    1    5
18050     6    2    5    1    1    6
18100     7    1    5    1    1    7
18150     8    2    6    1    1    8
18200     9    2    4    1    1    9
18250    10    3    5    1    1   10
18300
18350    2   1  2.0  2.0                           -2.   -2.
18400    3   1  2.0  2.0                            -2.  -2.
18450    4   1  2.0  2.0                           -2.   -2.
18500    5   1  2.0  2.0                            -2.  -2.
18550
18600     2    1           -100000.
18650     3    1           -100000.
18700
18800 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
18850
```

Echo of Input Cards for the First Stage of Design ($\alpha = 0$)

STRESS & DISP. CONSTRAINTS - 10 BAR CANTILEVER - VENKAYYA,S EX.4,CASE. 2

NUMBER OF NODAL POINTS = 6
NUMBER OF ELEMENT TYPES = 1
NUMBER OF LOAD CASES = 1
NUMBER OF DES. VARIABLES = 10

DESIGN CONTROL DATA

NCYCL = 1
KSCALE= 1
DELTA = 0.1000E-01
EPSIL = 0.1000E-01
KDISP = 1
OMEGA = 0.80000
ALPA = 0.00000

DESIGN VARIABLE INPUT DATA

DESIGN VARIABLE NUMBER	INITIAL VALUE	MIN ALLOWABLE VALUE
1	0.2000E 02	0.1000E 00
2	0.2000E 02	0.1000E 00
3	0.2000E 02	0.1000E 00
4	0.2000E 02	0.1000E 00
5	0.2000E 02	0.1000E 00
6	0.2000E 02	0.1000E 00
7	0.2000E 02	0.1000E 00
8	0.2000E 02	0.1000E 00
9	0.2000E 02	0.1000E 00
10	0.2000E 02	0.1000E 00

NODAL POINT INPUT DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	/------NODAL POINT COORDINATES--------/ X	Y	Z		T
1	1	1	-1	-1	-1	-1	0.000	0.000	0.000	0 ,	0.000
2	0	0	0	0	0	0	360.000	0.000	0.000	0	0.000
3	0	0	0	0	0	0	720.000	0.000	0.000	0	0.000
4	0	0	0	0	0	0	720.000	360.000	0.000	0	0.000
5	0	0	0	0	0	0	360.000	360.000	0.000	0	0.000
6	1	1	1	1	1	1	0.000	360.000	0.000	0	0.000

GENERATED NODAL DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	/------NODAL POINT COORDINATES--------/ X	Y	Z	T
1	1	1	-1	-1	-1	-1	0.000	0.000	0.000	0.000
2	0	0	-1	-1	-1	-1	360.000	0.000	0.000	0.000
3	0	0	-1	-1	-1	-1	720.000	0.000	0.000	0.000
4	0	0	-1	-1	-1	-1	720.000	360.000	0.000	0.000
5	0	0	-1	-1	-1	-1	360.000	360.000	0.000	0.000

Computer Printout for the First Stage of Design ($\alpha = 0$)
(Input data and the initial design only are reproduced.)

```
6    1    1    1    1    1    1      0.000     360.000      0.000      0.000
```

EQUATION NUMBERS

N	X	Y	Z	XX	YY	ZZ
1	0	0	0	0	0	0
2	1	2	0	0	0	0
3	3	4	0	0	0	0
4	5	6	0	0	0	0
5	7	8	0	0	0	0
6	0	0	0	0	0	0

NUMBER OF TRUSS ELEMENTS = 10
CONSTRUCTION CODE = 1
NUMBER OF MATERIALS = 1
NUMBER OF TEMPS FOR WHICH MATL PROPS GIVEN= 1
NUMBER OF DIFFERENT GEOMETRIES PROPS GIVEN= 1

MATERIAL PROPERTY CARDS

MATERIAL NUMBER	NUMBER OF TEMPS	SPECIFIC WEIGHT	TEMP	YOUNGS MODULUS	COEFFT OF THERM EXPAN	/--ALLOWABLE STRESSES--/ TENSION	COMPRESSION
1	1	0.1000E 00	0.0000E 00	0.1000E 08	0.0000E 00	0.2500E 05	0.2500E 05

GEOMETRIC PROPERTY CARDS

GEOMETRY NUMBER	X-SECT AREA	/--MOMENTS OF INERTIA--/ YY	ZZ
1	0.1000D 01	0.1000E 07	0.1000E 07

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.0000000D 00	0.0000000D 00	0.0000000D 00	0.0000000D 00
Y-DIR	0.0000000D 00	0.0000000D 00	0.0000000D 00	0.0000000D 00
Z-DIR	0.0000000D 00	0.0000000D 00	0.0000000D 00	0.0000000D 00
TEMP	0.0000000D 00	0.0000000D 00	0.0000000D 00	0.0000000D 00

PROCESSED ELEMENT DATA

ELEMENT NUMBER	/-NODE NOS-/ I	J	/--ELEMENT ID NOS-/ MATL	GEOMY	D VAR	DESIGN VAR FRACTION	REFERENCE TEMP	END FIXITY COEFFICIENTS YY	ZZ	BAND WIDTH
1	1	2	1	1	1	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	2
2	2	3	1	1	2	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	4
3	3	4	1	1	3	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	4
4	4	5	1	1	4	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	4
5	5	6	1	1	5	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	2
6	2	5	1	1	6	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	8
7	1	5	1	1	7	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	2
8	2	6	1	1	8	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	2
9	2	4	1	1	9	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	6
10	3	5	1	1	10	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	6

STRUCTURE LOAD CASE	STRUCTURE LOAD MULTIPLIERS A	B	C	D
1	0.000	0.000	0.000	0.000

NODAL DISPLACEMENT/ROTATION CONSTRAINTS

NODE NO.	LOAD CASE	/----MAX.ALLOWABLE DISPLACEMENTS AND ROTATIONS---- DX	DY	DZ	RX	RY	RZ	-DX	-DY	-DZ	-RX	-RY	-RZ ---/
2	1	2.00000	2.00000	0.00000	0.00000	0.00000	0.00000	-2.00000	-2.00000	0.00000	0.00000	0.00000	0.00000
3	1	2.00000	2.00000	0.00000	0.00000	0.00000	0.00000	-2.00000	-2.00000	0.00000	0.00000	0.00000	0.00000

4	1	2.00000	2.00000	0.00000	0.00000	0.00000	0.00000	-2.00000	-2.00000	0.00000	0.00000	0.00000	0.00000
5	1	2.00000	2.00000	0.00000	0.00000	0.00000	0.00000	-2.00000	-2.00000	0.00000	0.00000	0.00000	0.00000

NODAL POINT LOADS

NODE NO.	LOAD CASE	APPLIED LOADS RX	RY	RZ	MX	MY	MZ
2	1	0.000E 00	-0.100E 06	0.000E 00	0.000E 00	0.000E 00	0.000E 00
3	1	0.000E 00	-0.100E 06	0.000E 00	0.000E 00	0.000E 00	0.000E 00

TOTAL NUMBER OF EQUATIONS = 8
BANDWIDTH = 8
NUMBER OF EQUATIONS IN A BLOCK = 8
NUMBER OF BLOCKS = 1

****
ANALYSIS OF DESIGN NUMBER 0
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
6	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
5	1	3.517E-01	-8.372E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
4	1	4.239E-01	-1.898E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
3	1	-4.761E-01	-1.970E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	1	-3.683E-01	-9.011E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
1	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.2000E 02	0.2000E 02	0.2000E 02	0.2000E 02	0.2000E 02	0.2000E 02	0.2000E 02	0.2000E 02	0.2000E 02	0.2000E 02

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT	X-SECT AREA	LOAD COND	AXIAL FORCE
1	0.2000E 02	1	-0.2046E 06
2	0.2000E 02	1	-0.5988E 05
3	0.2000E 02	1	0.4012E 05
4	0.2000E 02	1	0.4012E 05
5	0.2000E 02	1	0.1954E 06
6	0.2000E 02	1	0.3549E 05
7	0.2000E 02	1	-0.1349E 06
8	0.2000E 02	1	0.1480E 06
9	0.2000E 02	1	-0.5674E 05
10	0.2000E 02	1	0.8468E 05

****
EVALUATION OF DESIGN NUMBER 0
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.4093E 00	1	1
MIN	0.7098E-01	1	6

MAX DISP RATIOS	LOAD COND	EQN NUMBER
-0.9849E 00	1	4
-0.9488E 00	1	6

UNIFORM SCALING OPERATION FOLLOWS

SCALE FACTOR IS 0.985AND DETERMINED BY DISPLACEMENT CONSTRAINTS

DESIGN VARIABLES OF SCALED (CRITICAL) DESIGN ARE

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.1970E 02	0.1970E 02	0.1970E 02	0.1970E 02	0.1970E 02	0.1970E 02	0.1970E 02	0.1970E 02	0.1970E 02	0.1970E 02

STRUCTURAL WEIGHT= 0.8266E 04

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR DISPT.   CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	ACT	-0.15343E 01
2	ACT	-0.16600E 00
3	ACT	-0.90133E-01
4	ACT	-0.90135E-01
5	ACT	-0.14767E 01
6	PASS	0.82555E-02
7	ACT	-0.46441E 00
8	ACT	-0.51878E 00
9	ACT	-0.18027E 00
10	ACT	-0.33199E 00

NO. OF ACTIVE DISPLACEMENT CONSTRAINTS ARE   1

****
ANALYSIS OF DESIGN NUMBER 5
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
6	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
5	1	2.379E-01	-7.369E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
4	1	2.344E-01	-2.004E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
3	1	-5.402E-01	-2.003E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	1	-2.995E-01	-1.478E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
1	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.2379E 02	0.1496E 02	0.3181E 00	0.1000E 00	0.3058E 02	0.1000E 00	0.2083E 02	0.8460E 01	0.1188E 00	0.2088E 02

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT	X-SECT AREA	LOAD COND	AXIAL FORCE
1	0.2379E 02	1	-0.1979E 06
2	0.1496E 02	1	-0.1000E 06
3	0.3181E 00	1	-0.9658E 01
4	0.1000E 00	1	-0.9672E 01
5	0.3058E 02	1	0.2021E 06
6	0.1000E 00	1	0.2059E 04
7	0.2083E 02	1	-0.1443E 06
8	0.8460E 01	1	0.1385E 06
9	0.1188E 00	1	0.1367E 02
10	0.2088E 02	1	0.1414E 06

****
EVALUATION OF DESIGN NUMBER 5
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.1000E 01	0	4
MIN	0.2643E 00	1	5

MAX DISP RATIOS	LOAD COND	EQN NUMBER
-0.1001E 01	1	4
-0.1002E 01	1	6

Computer Printout for the Second Stage of Design ($\alpha = 0.25$)
(The last two designs only are reproduced.)

DESIGN IS CRITICAL

STRUCTURAL WEIGHT= 0.5074E 04

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR DISPT. CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	ACT	-0.96269E 00
2	ACT	-0.10121E 01
3	ACT	-0.21610E 00
4	PASS	0.22408E 00
5	ACT	-0.10230E 01
6	PASS	0.52923E 01
7	ACT	-0.10535E 01
8	ACT	-0.70438E 00
9	PASS	0.31737E 00
10	ACT	-0.10386E 01

NO. OF ACTIVE DISPLACEMENT CONSTRAINTS ARE 1

****
ANALYSIS OF DESIGN NUMBER 6
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
6	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
5	1	2.352E-01	-7.205E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
4	1	1.550E-01	-2.037E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
3	1	-5.454E-01	-1.576E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	1	-3.063E-01	-1.803E 00	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
1	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.2313E 02	0.1509E 02	0.1311E 00	0.1000E 00	0.3111E 02	0.1000E 00	0.2166E 02	0.6584E 01	0.1000E 00	0.2148E 02

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT	X-SECT AREA	LOAD COND	AXIAL FORCE
1	0.2313E 02	1	-0.1968E 06
2	0.1509E 02	1	-0.1002E 06
3	0.1311E 00	1	-0.2229E 03
4	0.1000E 00	1	-0.2229E 03
5	0.3111E 02	1	0.2032E 06
6	0.1000E 00	1	0.3007E 04
7	0.2166E 02	1	-0.1460E 06
8	0.6584E 01	1	0.1369E 06
9	0.1000E 00	1	0.3152E 03
10	0.2148E 02	1	0.1417E 06

****
EVALUATION OF DESIGN NUMBER 6
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.1203E 01	1	6
MIN	0.2613E 00	1	5

MAX DISP RATIOS	LOAD COND	ECN NUMBER
-0.9880E 00	1	4
-0.1019E 01	1	6

```
UNIFORM SCALING OPERATION FOLLOWS

SCALE FACTOR IS  1.203AND DETERMINED BY STRESS CONSTRAINTS

DESIGN VARIABLES OF SCALED (CRITICAL) DESIGN ARE

VALUES OF DESIGN VARIABLES
```

	1	2	3	4	5	6	7	8	9	10
0	0.2781E 02	0.1815E 02	0.1576E 00	0.1203E 00	0.3741E 02	0.1203E 00	0.2605E 02	0.7919E 01	0.1203E 00	0.2584E 02

```
STRUCTURAL WEIGHT= 0.6067E 04

REDESIGN OPERATION FOLLOWS
TERMINAL DESIGN---LIGHTEST CRITICAL DESIGN IS DESIGN NUMBER   5
```

M. BEAM ELEMENTS

M.1 Plane Rectangular Frame

The geometry of the frame is defined in Fig. M.1.1. Each of the twelve elements is sized independently, but the cross-sectional proportions of the reference section must be maintained throughout the structure, i.e., Construction Code No. 2 is to be used.

The frame is subjected to three load conditions shown in Fig. M.1.2. The remainder of the design data is:

$E = 29 \times 10^6$ psi (Young's modulus).

$\sigma_t^* = \sigma_c^* = 29{,}000$ psi (allowable stresses).

$\rho = 0.283$ lb/in^3 (specific weight).

$A^* = 0.1$ in^2 for all elements (min. allowable cross-sectional areas). Since this value is never reached during design, it is equivalent to having no lower bound on the element sizes.

$A = 30.0$ in^2 for all elements. (initial cross-sectional areas). Note that this value differs from the cross-sectional area of the reference section.

$u_x^* = \pm\ 0.3$ in for nodes 1, 5, 6 and 10 (max. allowable displacements in $\pm$ x-direction).

Symmetry of structural layout, loading and the constraints will result in a material distribution that is also symmetric. We may take advantage of this knowledge and impose symmetry conditions on the

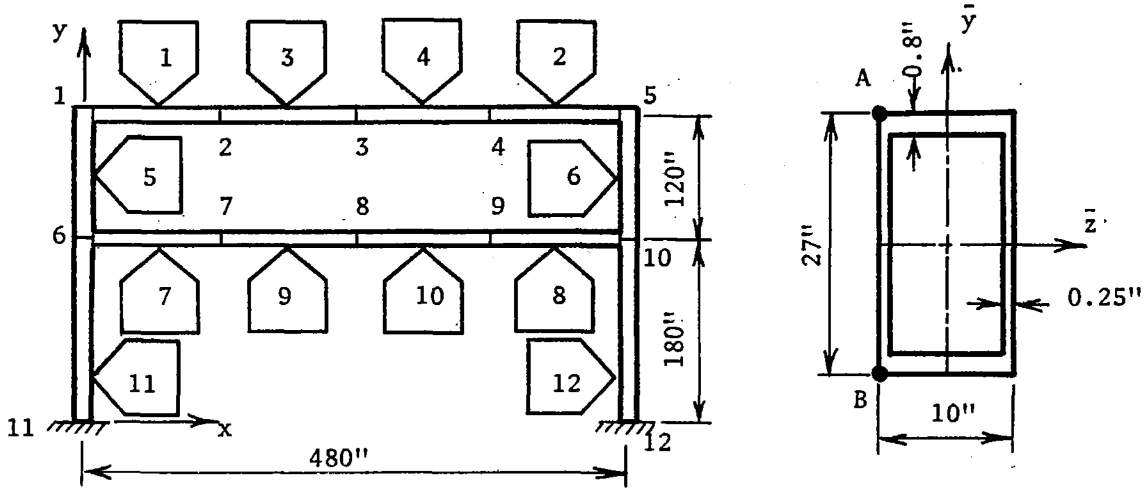

Figure M.1.1

Layout of Elements and Reference Cross Section

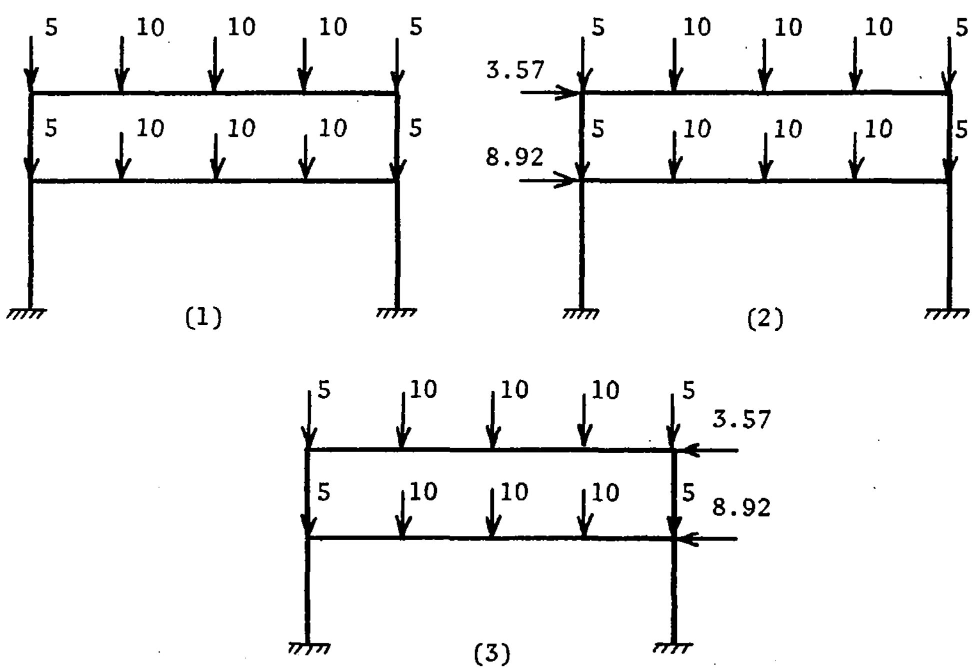

Figure M.1.2

Load Conditions (loads in kips)

design at the outset, using the following equal size constraints:

$$A_1 = A_2 \ (= D_1) \ , \qquad A_3 = A_4 \ (= D_2) \ ,$$

$$A_5 = A_6 \ (= D_3) \ , \qquad A_7 = A_8 \ (= D_4) \ ,$$

$$A_9 = A_{10} \ (= D_5) \ , \qquad A_{11} \doteq A_{12} \ (= D_6) \ ,$$

where D_i, $i = 1,2,\ldots,6$ are the independent design variables. It is now sufficient to consider load conditions (1) and (2) only.

The design history of the frame is summarized in Table M.1.1 The final design was reached after six redesign operations, but the weight changes in the last two redesigns are negligible. The design is governed by stress and displacement constraints simultaneously.

A frame with identical layout, loading and constraints has been treated in Ref. [9] by a somewhat different design method. The optimal design was reached in 12 design cycles. A direct comparison of the results with those of DESAP 1 is not practical, however, because the proportions of the cross section were not kept entirely constant in [9].

Special notes on input-output:

1) We set KSCALE = 2 in the Design Control Data,thereby implying that the internal forces remain unchanged upon uniform scaling, i.e. that scaling is an exact operation. The above is true only if the contribution of the axial deformations is neglected in comparison to the bending deformations, which may indeed be done for orthogonal frames. This approximation in no way impairs the accuracy of the

final design; it simply means that for a scaled design the maximum stress or displacement ratio (whichever governs) is not precisely one.

2) Node number 13 in Nodal Point Input Data is used solely for defining the direction of the local $\bar{y}$-axis of each element (also see Element Data).

3) Bending takes place about the local $\bar{z}$-axis; consequently properties of the cross section about $\bar{x}$ and $\bar{y}$-axes do not have to be defined on the Geometric Property Cards.

Des. Var.	Elem.	Critical, Scaled Designs (sq. in.)						
		0	1	2	3	4	5	6
1	1,2	30,00	24.39	19.91	16.30	13.84	12.51	11.70
2	3,4	30.00	21.05	15.31	11.68	9.57	8.55	8.09
3	5,6	30.00	22.68	16.63	12.10	10.99	11.08	11.19
4	7,8	30.00	34.83	39.11	41.35	43.07	44.06	44.59
5	9,10	30.00	22.33	20.60	21.74	22.63	23.16	23.45
6	11,12	30.00	30.25	31.11	31.43	31.85	32.21	32.37
Wt. (kips)		13.24	11.59	10.75	10.21	10.04	10.03	10.03

Table M.1.1

Design History of Cross-Sectional Areas and Total Structural Weight.

```
00001 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
08700  SYMMETRICAL RECTANGULAR FRAME- STRESS AND DISP. CONSTS, VENKAYYA'S PAPER P.303
08750    13    1    2    6
08800    10         2 0.025     0.1          1    1    1  0.8       0.67
08850     6 30.         0.1
08900     1             -1   -1   -1                300.
08950     5                               480.      300.                1
09000     6                                         180.
09050    10                               480.      180.                1
09100    11    1    1
09150    12    1    1    1    1    1      480.
09200    13    1    1    1    1    1    1 -100.     600.
09250     2   12    1    1    2
09300     1      0.283    29000000.            29000.    29000.
09350     1    1 28.7                          3429.4
09400                      254.03                        -254.03
09450
09500
09550
09600     2    4    5   13    1    1    1                                            3
09650     4    3    4   13    1    1    2
09700     6   10    5   13    1    1    3                                            6
09750     8    9   10   13    1    1    4                                            3
09800    10    8    9   13    1    1    5
09850    11   11    6   13    1    1    6
09900    12   12   10   13    1    1    6
09950
10000
10050    1   10.3                                 0.3
10100    1   20.3                                 0.3
10150    5   10.3                                 0.3
10200    5   20.3                                 0.3
10250    6   10.3                                 0.3
10300    6   20.3                                 0.3
10350   10   10.3                                 0.3
10400   10   20.3                                 0.3
10450
10500     1    1           -5000.
10550     1    2 3570.     -5000.
10600     2    1          -10000.
10650     2    2          -10000.
10700     3    1          -10000.
10750     3    2          -10000.
10800     4    1          -10000.
10850     4    2          -10000.
10900     5    1           -5000.
10950     5    2           -5000.
11000     6    1           -10000.
11050     6    2 9920.     -10000.
11100     7    1           -20000.
11150     7    2           -20000.
11200     8    1           -20000.
11250     8    2           -20000.
11300     9    1           -20000.
11350     9    2           -20000.
11400    10    1           -10000.
11450    10    2           -10000.
11500
11550 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
11600
```

Echo of Input Cards

SYMMETRICAL RECTANGULAR FRAME- STRESS AND DISP. CONSTS, VENKAYYA'S PAPER P.303

NUMBER OF NODAL POINTS = 13
NUMBER OF ELEMENT TYPES = 1
NUMBER OF LOAD CASES = 2
NUMBER OF DES. VARIABLES = 6

DESIGN CONTROL DATA

NCYCL = 10
KSCALF= 2
DELTA = 0.2500E-01
EPSIL = 0.1000E 00
KDISP = 1
OMEGA = 0.80000
ALPA = 0.67000

DESIGN VARIABLE INPUT DATA

DESIGN VARIABLE NUMBER	INITIAL VALUE	MIN ALLOWABLE VALUE
1	0.3000E 02	0.1000E 00
2	0.3000E 02	0.1000E 00
3	0.3000E 02	0.1000E 00
4	0.3000E 02	0.1000E 00
5	0.3000E 02	0.1000E 00
6	0.3000E 02	0.1000E 00

NODAL POINT INPUT DATA

NODE	BOUNDARY CONDITION CODES						/------NODAL POINT COORDINATES--------/				
NUMBER	X	Y	Z	XX	YY	ZZ	X	Y	Z		T
1	0	0	-1	-1	-1	0	0.000	300.000	0.000	0	0.000
5	0	0	0	0	0	0	480.000	300.000	0.000	1	0.000
6	0	0	0	0	0	0	0.000	180.000	0.000	0	0.000
10	0	0	0	0	0	0	480.000	180.000	0.000	1	0.000
11	1	1	0	0	0	0	0.000	0.000	0.000	0	0.000
12	1	1	1	1	1	0	480.000	0.000	0.000	0	0.000
13	1	1	1	1	1	1	-100.000	600.000	0.000	0	0.000

GENERATED NODAL DATA

NODE	BOUNDARY CONDITION CODES						/------NODAL POINT COORDINATES--------/			
NUMBER	X	Y	Z	XX	YY	ZZ	X	Y	Z	T
1	0	0	-1	-1	-1	0	0.000	300.000	0.000	0.000
2	0	0	-1	-1	-1	0	120.000	300.000	0.000	0.000
3	0	0	-1	-1	-1	0	240.000	300.000	0.000	0.000
4	0	0	-1	-1	-1	0	360.000	300.000	0.000	0.000
5	0	0	-1	-1	-1	0	480.000	300.000	0.000	0.000
6	0	0	-1	-1	-1	0	0.000	180.000	0.000	0.000
7	0	0	-1	-1	-1	0	120.000	180.000	0.000	0.000
8	0	0	-1	-1	-1	0	240.000	180.000	0.000	0.000

Computer Printout
(Input data, the initial design and the final design only are reproduced.)

9	0	0	-1	-1	-1	0	360.000	180.000	0.000	0.000
10	0	0	-1	-1	-1	0	480.000	180.000	0.000	0.000
11	1	1	-1	-1	-1	0	0.000	0.000	0.000	0.000
12	1	1	1	1	1	0	480.000	0.000	0.000	0.000
13	1	1	1	1	1	1	-100.000	600.000	0.000	0.000

EQUATION NUMBERS

N	X	Y	Z	XX	YY	ZZ
1	1	2	0	0	0	3
2	4	5	0	0	0	6
3	7	8	0	0	0	9
4	10	11	0	0	0	12
5	13	14	0	0	0	15
6	16	17	0	0	0	18
7	19	20	0	0	0	21
8	22	23	0	0	0	24
9	25	26	0	0	0	27
10	28	29	0	0	0	30
11	0	0	0	0	0	31
12	0	0	0	0	0	32
13	0	0	0	0	0	0

THREE DIMENSIONAL BEAM ELEMENTS

NUMBER OF BEAM ELEMENTS	=	12
CONSTRUCTION CODE	=	2
NUMBER OF MATERIALS	=	1
NUMBER OF GEOMETRIC PROPERTIES	=	1
NUMBER OF FIXED-END FORCE SETS	=	0

MATERIAL PROPERTY CARDS

MATERIAL NUMBER	SPECIFIC WEIGHT	YOUNGS MODULUS	POISSONS RATIO	/--------ALLOWABLE STRESSES--------/ TENSION	COMPRESSION	SHEAR
1	0.2830D 00	0.2900E 08	0.0000E 00	0.2900E 05	0.2900E 05	0.1673E 05

GEOMETRIC PROPERTY CARDS

PROPERTY NUMBER	X-SECT KODE	X-SECT AREA	/------PROPERTIES OF X-SECTION-----/ X-AXIS	Y-AXIS	Z-AXIS	
1	1	0.2870D 02	0.0000E 00	0.0000E 00	0.3429E 04	MOMENTS OF INERTIA
			0.0000E 00	0.0000E 00	0.2540E 03	SECT MODULI FOR POINT A
			0.0000E 00	0.0000E 00	-0.2540E 03	SECT MODULI FOR POINT B

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.0000000D 00	0.0000000D 00	0.0000000D 00	0.0000000D 00
Y-DIR	0.0000000D 00	0.0000000D 00	0.0000000D 00	0.0000000D 00
Z-DIR	0.0000000D 00	0.0000000D 00	0.0000000D 00	0.0000000D 00

PROCESSED ELEMENT DATA

ELEMENT NUMBER	/---NODE NOS--/ I	J	K	/--ELEMENT ID NOS-/ MATL	GEOMY	D VAR	DESIGN VAR FRACTION	FIXED END-FORCE ID A	B	C	D	END RELEASE CODES I	J	BAND WIDTH
1	1	2	13	1	1	1	0.1000E 01	0	0	0	0	000000	000000	6
2	4	5	13	1	1	1	0.1000E 01	0	0	0	0	000000	000000	6
3	2	3	13	1	1	2	0.1000E 01	0	0	0	0	000000	000000	6
4	3	4	13	1	1	2	0.1000E 01	0	0	0	0	000000	000000	6
5	6	1	13	1	1	3	0.1000E 01	0	0	0	0	000000	000000	18
6	10	5	13	1	1	3	0.1000E 01	0	0	0	0	000000	000000	18
7	6	7	13	1	1	4	0.1000E 01	0	0	0	0	000000	000000	6
8	9	10	13	1	1	4	0.1000E 01	0	0	0	0	000000	000000	6
9	7	8	13	1	1	5	0.1000E 01	0	0	0	0	000000	000000	6
10	8	9	13	1	1	5	0.1000E 01	0	0	0	0	000000	000000	6
11	11	6	13	1	1	6	0.1000E 01	0	0	0	0	000000	000000	16
12	12	10	13	1	1	6	0.1000E 01	0	0	0	0	000000	000000	5

STRUCTURE LOAD CASE	STRUCTURE LOAD MULTIPLIERS A	B	C	D
1	0.000	0.000	0.000	0.000
2	0.000	0.000	0.000	0.000

NODAL DISPLACEMENT/ROTATION CONSTRAINTS

NODE NO.	LOAD/ CASE	DX	DY	DZ	RX	RY	RZ	-DX	-DY	-DZ	-RX	-RY	-RZ
		-----	-----	-----	-----	-----MAX.ALLOWABLE	DISPLACEMENTS AND ROTATIONS-----	-----	-----	-----	-----	-----	-----/
1	1	0.30000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.30000	0.00000	0.00000	0.00000	0.00000	0.00000
1	2	0.30000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.30000	0.00000	0.00000	0.00000	0.00000	0.00000
5	1	0.30000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.30000	0.00000	0.00000	0.00000	0.00000	0.00000
5	2	0.30000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.30000	0.00000	0.00000	0.00000	0.00000	0.00000
6	1	0.30000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.30000	0.00000	0.00000	0.00000	0.00000	0.00000
6	2	0.30000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.30000	0.00000	0.00000	0.00000	0.00000	0.00000
10	1	0.30000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.30000	0.00000	0.00000	0.00000	0.00000	0.00000
10	2	0.30000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.30000	0.00000	0.00000	0.00000	0.00000	0.00000

NODAL POINT LOADS

NODE NO.	LOAD CASE	RX	RY	RZ	MX	MY	MZ
		APPLIED LOADS					
1	1	0.000E 00	-0.500E 04	0.000E 00	0.000E 00	0.000E 00	0.000E 00
1	2	0.357E 04	-0.500E 04	0.000E 00	0.000E 00	0.000E 00	0.000E 00
2	1	0.000E 00	-0.100E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
2	2	0.000E 00	-0.100E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
3	1	0.000E 00	-0.100E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
3	2	0.000E 00	-0.100E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
4	1	0.000E 00	-0.100E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
4	2	0.000E 00	-0.100E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
5	1	0.000E 00	-0.500E 04	0.000E 00	0.000E 00	0.000E 00	0.000E 00
5	2	0.000E 00	-0.500E 04	0.000E 00	0.000E 00	0.000E 00	0.000E 00
6	1	0.000E 00	-0.100E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
6	2	0.892E 04	-0.100E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
7	1	0.000E 00	-0.200E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
7	2	0.000E 00	-0.200E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
8	1	0.000E 00	-0.200E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
8	2	0.000E 00	-0.200E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
9	1	0.000E 00	-0.200E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
9	2	0.000E 00	-0.200E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
10	1	0.000E 00	-0.100E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00
10	2	0.000E 00	-0.100E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00

```
TOTAL NUMBER OF EQUATIONS      =   32
BANDWIDTH                      =   18
NUMBER OF EQUATIONS IN A BLOCK =   32
NUMBER OF BLOCKS               =    1
```

****
ANALYSIS OF DESIGN NUMBER 0
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
13	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
	2	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
12	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	-3.0391E-04
	2	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	-1.8998E-03
11	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	3.0392E-04
	2	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	-1.3124E-03
10	1	5.740E-03	-1.241E-02	0.000E-01	0.0000E-01	0.0000E-01	5.1216E-04
	2	2.378E-01	-1.357E-02	0.000E-01	0.0000E-01	0.0000E-01	-1.6378E-04
9	1	2.869E-03	-1.778E-01	0.000E-01	0.0000E-01	0.0000E-01	1.5812E-03
	2	2.355E-01	-1.481E-01	0.000E-01	0.0000E-01	0.0000E-01	1.6598E-03
8	1	-5.849E-07	-2.859E-01	0.000E-01	0.0000E-01	0.0000E-01	6.8365E-10
	2	2.332E-01	-2.858E-01	0.000E-01	0.0000E-01	0.0000E-01	3.3033E-04
7	1	-2.871E-03	-1.778E-01	0.000E-01	0.0000E-01	0.0000E-01	-1.5812E-03
	2	2.310E-01	-2.073E-01	0.000E-01	0.0000E-01	0.0000E-01	-1.5018E-03
6	1	-5.741E-03	-1.241E-02	0.000E-01	0.0000E-01	0.0000E-01	-5.1215E-04
	2	2.287E-01	-1.126E-02	0.000E-01	0.0000E-01	0.0000E-01	-1.1864E-03
5	1	-7.251E-03	-1.517E-02	0.000E-01	0.0000E-01	0.0000E-01	2.8481E-04
	2	2.876E-01	-1.657E-02	0.000E-01	0.0000E-01	0.0000E-01	-4.2769E-05
4	1	-3.626E-03	-1.004E-01	0.000E-01	0.0000E-01	0.0000E-01	8.0498E-04
	2	2.915E-01	-8.636E-02	0.000E-01	0.0000E-01	0.0000E-01	8.3516E-04
3	1	-8.122E-07	-1.554E-01	0.000E-01	0.0000E-01	0.0000E-01	-8.6416E-10
	2	2.954E-01	-1.546E-01	0.000E-01	0.0000E-01	0.0000E-01	1.5167E-04
2	1	3.624E-03	-1.004E-01	0.000E-01	0.0000E-01	0.0000E-01	-8.0498E-04
	2	2.993E-01	-1.133E-01	0.000E-01	0.0000E-01	0.0000E-01	-7.6807E-04
1	1	7.249E-03	-1.517E-02	0.000E-01	0.0000E-01	0.0000E-01	-2.8481E-04
	2	3.031E-01	-1.377E-02	0.000E-01	0.0000E-01	0.0000E-01	-5.9891E-04

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.3000E 02	0.3000E 02	0.3000E 02	0.3000E 02	0.3000E 02	0.3000E 02				

ANALYSIS OF BEAM ELEMENTS, CONSTRN CODE= 2

ELEMENT	X-SECT AREA	LOAD COND	AXIAL PX	SHEAR PY	SHEAR PZ	TORQUE MX	MOMENT MY	MOMENT MZ
1	0.3000E 02	1	0.2628E 05	0.1500E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1371E 07
			-0.2628E 05	-0.1500E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.4289E 06
		2	0.2821E 05	0.1322E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.9462E 06
			-0.2821E 05	-0.1322E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.6398E 06
2	0.3000E 02	1	0.2628E 05	-0.1500E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.4290E 06
			-0.2628E 05	0.1500E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.1371E 07
		2	0.2821E 05	-0.1678E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.2120E 06
			-0.2921E 05	0.1678E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.1802E 07
3	0.3000E 02	1	0.2628E 05	0.5000E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.4289E 06
			-0.2628E 05	-0.5000E 04	0.0000E 00	0.0000E 00	0.0000E 00	0.1029E 07

		2	0.2821E 05	0.3217E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.6398E 06
			-0.2821E 05	-0.3217E 04	0.0000E 00	0.0000E 00	0.0000E 00	0.1076E 07
4	0.3000E 02	1	0.2628E 05	-0.5000E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.1029E 07
			-0.2628E 05	0.5000E 04	0.0000E 00	0.0000E 00	0.0000E 00	0.4290E 06
		2	0.2820E 05	-0.6783E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.1026E 07
			-0.2820E 05	0.6783E 04	0.0000E 00	0.0000E 00	0.0000E 00	0.2120E 06
5	0.3000E 02	1	0.2000E 05	-0.2628E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.1783E 07
			-0.2000E 05	0.2628E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.1371E 07
		2	0.1822E 05	-0.2464E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.2010E 07
			-0.1822E 05	0.2464E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.9462E 06
6	0.3000E 02	1	0.2000E 05	0.2628E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1783E 07
			-0.2000E 05	-0.2628E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1371E 07
		2	0.2178E 05	0.2821E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1583E 07
			-0.2178E 05	-0.2821E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1802E 07
7	0.3000E 02	1	-0.2081E 05	0.3000E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.2768E 07
			0.2081E 05	-0.3000E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.8319E 06
		2	-0.1656E 05	0.2621E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1858E 07
			0.1656E 05	-0.2621E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1287E 07
8	0.3000E 02	1	-0.2081E 05	-0.3000E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.8319E 06
			0.2081E 05	0.3000E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.2768E 07
		2	-0.1656E 05	-0.3379E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.3763E 06
			0.1656E 05	0.3379E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.3679E 07
9	0.3000E 02	1	-0.2081E 05	0.1000E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.8319E 06
			0.2081E 05	-0.1000E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.2032E 07
		2	-0.1656E 05	0.6207E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.1287E 07
			0.1656E 05	-0.6207E 04	0.0000E 00	0.0000E 00	0.0000E 00	0.2032E 07
10	0.3000E 02	1	-0.2081E 05	-0.1000E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.2032E 07
			0.2081E 05	0.1000E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.8319E 06
		2	-0.1656E 05	-0.1379E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.2032E 07
			0.1656E 05	0.1379E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.3763E 06
11	0.3000E 02	1	0.6000E 05	-0.5474E 04	0.0000E 00	0.0000E 00	0.0000E 00	0.1822E 01
			-0.6000E 05	0.5474E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.9853E 06
		2	0.5442E 05	0.8451E 03	0.0000E 00	0.0000E 00	0.0000E 00	-0.9126E 01
			-0.5442E 05	-0.8451E 03	0.0000E 00	0.0000E 00	0.0000E 00	0.1521E 06
12	0.3000E 02	1	0.6000E 05	0.5474E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.1210E-01
			-0.6000E 05	-0.5474E 04	0.0000E 00	0.0000E 00	0.0000E 00	0.9853E 06
		2	0.6558E 05	0.1165E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.5388E 00
			-0.6558E 05	-0.1165E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.2096E 07

****
EVALUATION OF DESIGN NUMBER 0
****

	STRESS AREA RATIO	LOAD COND	DES VARIABLE
MAX	0.6153E 00	2	4
MIN	0.2791E 00	2	2

MAX	DISP RATIO	LOAD COND	FCN NUMBER

0.9586E 00	2	13
0.1010E 01	2	1

DESIGN IS CRITICAL

STRUCTURAL WEIGHT= 0.1324E 05

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR DISPT.   CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	ACT	-0.43333E 00
2	ACT	-0.95864E-01
3	ACT	-0.26092E 00
4	ACT	-0.14879E 01
5	ACT	-0.22564E 00
6	ACT	-0.10248E 01

NO. OF ACTIVE DISPLACEMENT CONSTRAINTS ARE   1

****
ANALYSIS OF DESIGN NUMBER 6
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
13	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
	2	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
12	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	-5.2431E-04
	2	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	-1.8670E-03
11	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	5.2430E-04
	2	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	-8.3316E-04
10	1	2.822E-03	-1.151E-02	0.000E-01	0.0000E-01	0.0000E-01	1.0016E-03
	2	1.967E-01	-1.258E-02	0.000E-01	0.0000E-01	0.0000E-01	4.5657E-04
9	1	1.849E-03	-1.931E-01	0.000E-01	0.0000E-01	0.0000E-01	1.7245E-03
	2	1.961E-01	-1.595E-01	0.000E-01	0.0000E-01	0.0000E-01	1.6410E-03
8	1	4.439E-07	-3.182E-01	0.000E-01	0.0000E-01	0.0000E-01	2.0773E-09
	2	1.950E-01	-3.190E-01	0.000E-01	0.0000E-01	0.0000E-01	4.7018E-04
7	1	-1.849E-03	-1.931E-01	0.000E-01	0.0000E-01	0.0000E-01	-1.7245E-03
	2	1.940E-01	-2.277E-01	0.000E-01	0.0000E-01	0.0000E-01	-1.8161E-03
6	1	-2.821E-03	-1.151E-02	0.000E-01	0.0000E-01	0.0000E-01	-1.0016E-03
	2	1.934E-01	-1.044E-02	0.000E-01	0.0000E-01	0.0000E-01	-1.5570E-03
5	1	-1.931E-02	-1.890E-02	0.000E-01	0.0000E-01	0.0000E-01	2.9196E-03
	2	2.585E-01	-2.013E-02	0.000E-01	0.0000E-01	0.0000E-01	2.3491E-03
4	1	-1.145E-02	-7.916E-01	0.000E-01	0.0000E-01	0.0000E-01	7.8155E-03
	2	2.670E-01	-7.616E-01	0.000E-01	0.0000E-01	0.0000E-01	7.8037E-03
3	1	8.358E-07	-1.352E 00	0.000E-01	0.0000E-01	0.0000E-01	1.3530E-08
	2	2.794E-01	-1.352E 00	0.000E-01	0.0000E-01	0.0000E-01	3.8323E-04
2	1	1.145E-02	-7.916E-01	0.000E-01	0.0000E-01	0.0000E-01	-7.8155E-03
	2	2.917E-01	-8.219E-01	0.000E-01	0.0000E-01	0.0000E-01	-7.8295E-03
1	1	1.931E-02	-1.890E-02	0.000E-01	0.0000E-01	0.0000E-01	-2.9196E-03
	2	3.002E-01	-1.767E-02	0.000E-01	0.0000E-01	0.0000E-01	-3.4933E-03

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.1179E 02	0.8091E 01	0.1119E 02	0.4459E 02	0.2345E 02	0.3237E 02				

ANALYSIS OF BEAM ELEMENTS, CONSTRN CODE= 2

ELEMENT	X-SECT AREA	LOAD COND	AXIAL RX	SHEAR RY	SHEAR RZ	TORQUE MX	MOMENT MY	MOMENT MZ
1	0.1179E 02	1	0.2239E 05	0.1500E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1585E 07
			-0.2239E 05	-0.1500E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.2148E 06
		2	0.2418E 05	0.1457E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1481E 07
			-0.2418E 05	-0.1457E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.2670E 06
2	0.1179E 02	1	0.2239E 05	-0.1500E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.2148E 06
			-0.2239E 05	0.1500E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.1585E 07
		2	0.2418E 05	-0.1543E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.1627E 06
			-0.2418E 05	0.1543E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.1690E 07
3	0.8091E 01	1	0.2239E 05	0.5000E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.2148E 06
			-0.2239E 05	-0.5000E 04	0.0000E 00	0.0000E 00	0.0000E 00	0.8148E 06

		2	0.2418E 05	0.4565E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.2670E 06
			-0.2418E 05	-0.4565E 04	0.0000E 00	0.0000E 00	0.0000E 00	0.8148E 06
4	0.8091E 01	1	0.2239E 05	-0.5000E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.8148E 06
			-0.2239E 05	0.5000E 04	0.0000E 00	0.0000E 00	0.0000E 00	0.2148E 06
		2	0.2418E 05	-0.5435E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.8148E 06
			-0.2418E 05	0.5435E 04	0.0000E 00	0.0000E 00	0.0000E 00	0.1627E 06
5	0.1119E 02	1	0.2000E 05	-0.2239E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.1102E 07
			-0.2000E 05	0.2239E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.1585E 07
		2	0.1957E 05	-0.2061E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.9926E 06
			-0.1957E 05	0.2061E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.1481E 07
6	0.1119E 02	1	0.2000E 05	0.2239E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1102E 07
			-0.2000E 05	-0.2239E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1585E 07
		2	0.2043E 05	0.2418E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1212E 07
			-0.2043E 05	-0.2418E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1690E 07
7	0.4459E 02	1	-0.1048E 05	0.3000E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.3246E 07
			0.1048E 05	-0.3000E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.3539E 06
		2	-0.6041E 04	0.2486E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.2010E 07
			0.6041E 04	-0.2486E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.9731E 06
8	0.4459E 02	1	-0.1048E 05	-0.3000E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.3539E 06
			0.1048E 05	0.3000E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.3246E 07
		2	-0.6042E 04	-0.3514E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.2609E 06
			0.6042E 04	0.3514E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.4478E 07
9	0.2345E 02	1	-0.1048E 05	0.1000E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.3539E 06
			0.1048E 05	-0.1000E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.1554E 07
		2	-0.6042E 04	0.4858E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.9731E 06
			0.6042E 04	-0.4858E 04	0.0000E 00	0.0000E 00	0.0000E 00	0.1556E 07
10	0.2345E 02	1	-0.1048E 05	-0.1000E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.1554E 07
			0.1048E 05	0.1000E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.3539E 06
		2	-0.6041E 04	-0.1514E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.1556E 07
			0.6041E 04	0.1514E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.2609E 06
11	0.3237E 02	1	0.6000E 05	-0.1191E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.2996E 01
			-0.6000E 05	0.1191E 05	0.0000E 00	0.0000E 00	0.0000E 00	-0.2144E 07
		2	0.5442E 05	-0.5651E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.6573E 01
			-0.5442E 05	0.5651E 04	0.0000E 00	0.0000E 00	0.0000E 00	-0.1017E 07
12	0.3237E 02	1	0.6000E 05	0.1191E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.2022E 01
			-0.6000E 05	-0.1191E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.2144E 07
		2	0.6558E 05	0.1814E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.2105E 00
			-0.6558E 05	-0.1814E 05	0.0000E 00	0.0000E 00	0.0000E 00	0.3266E 07

****
EVALUATION OF DESIGN NUMBER 6
****

	STRESS AREA RATIO	LOAD COND	DES VARIABLE
MAX	0.1003E 01	2	3
MIN	0.4290E 00	2	5

MAX DISP RATIO LOAD COND ECN NUMBER

```
        0.8617E 00         2         13
        0.1001E 01         2          1


DESIGN IS CRITICAL



STRUCTURAL WEIGHT= 0.1003E 05


REDESIGN OPERATION FOLLOWS


OPTIMALITY INDEX OF DESIGN VARIABLES FOR DISPT.   CONSTRAINTS

    DV NO ACT/PAS     INDEX

        1      ACT    -0.85546E 00
        2      ACT    -0.92101E 00
        3      PASS   -0.63051E 00
        4      ACT    -0.10200E 01
        5      ACT    -0.10212E 01
        6      ACT    -0.10075E 01

NO. OF ACTIVE DISPLACEMENT CONSTRAINTS ARE     1
DISPLACEMENT-CRITICAL DESIGN HAS CONVERGED
```

N. PLANE STRESS ELEMENTS

N.1 Rectangular Plate with Reinforced Hole

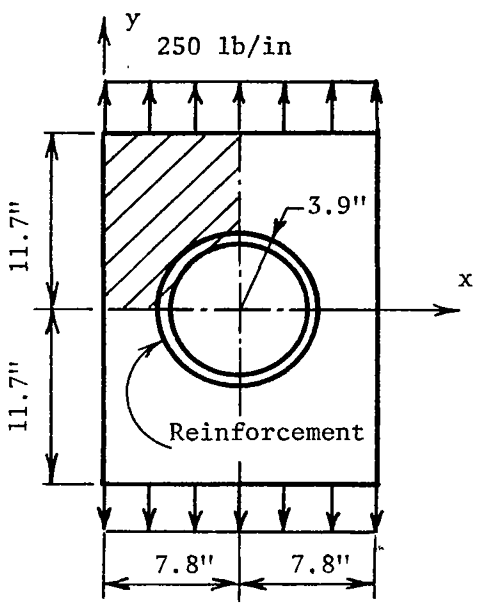

Figure N.1.1

Plate with Reinforced Hole

The plate under consideration is shown in Fig. N.1.1. Due to double symmetry, it is sufficient to treat one quadrant of the plate only, as indicated by the finite element mesh in Fig. N.1.2. Isotropic, plane stress elements (Construction Code No. 2) are used to model the plate, whereas the reinforcement is approximated by a series of bar elements (Construction Code No. 1 was chosen arbitrarily). Each element is sized independently.

Identical material is used for the plate and the reinforcement, with:

$E = 10^6$ psi (Young's modulus),

$\nu = 0.25$ (Poisson's ratio),

$\sigma_t^* = \sigma_c^* = 25{,}000$ psi (allowable stress),

$\rho = 0.1$ lb/cu.in. (specific weight).

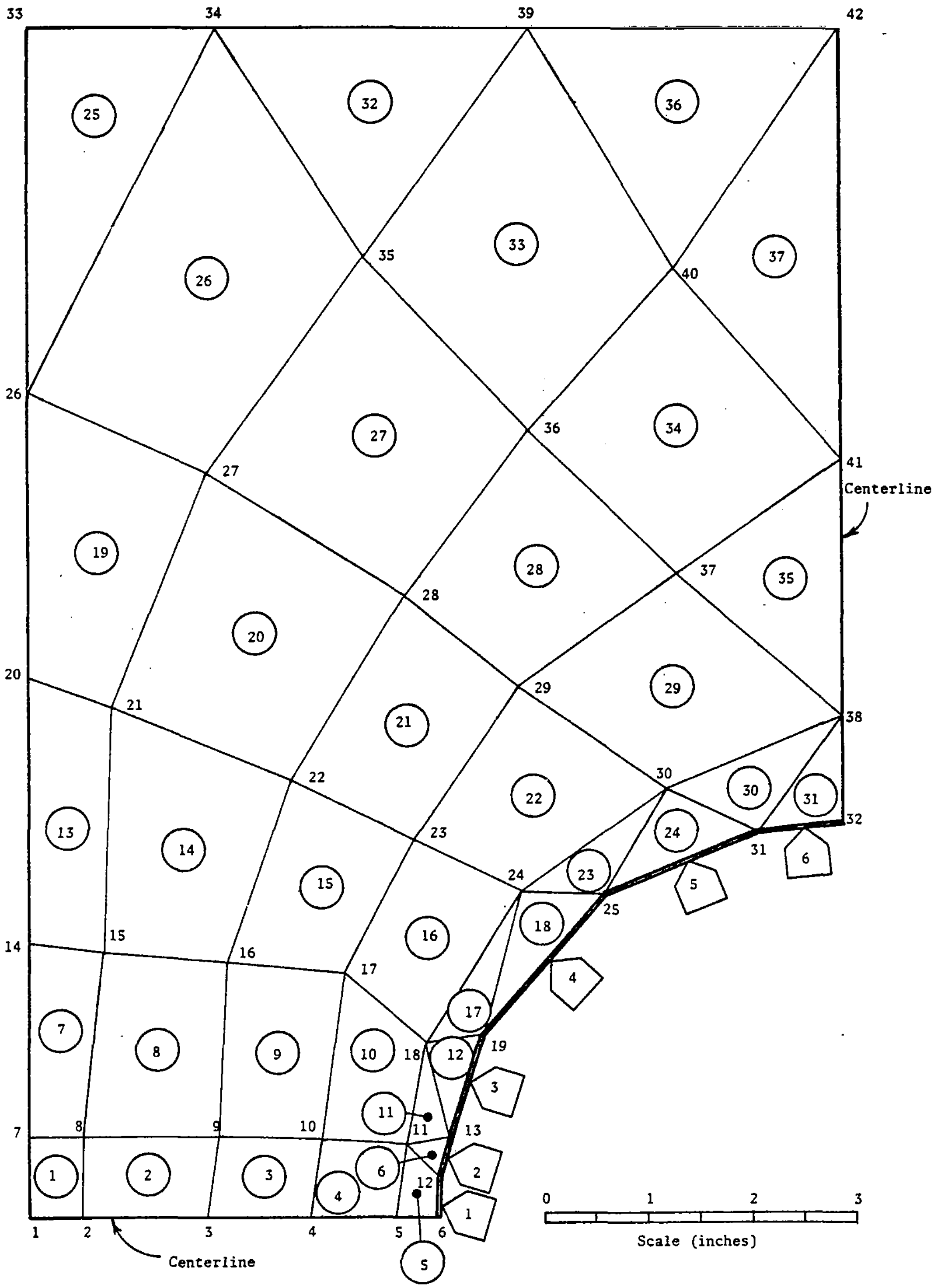

Figure N.1.2

Finite Element Mesh for a Quadrant of Plate

In addition to stress constraints, upper bounds are placed on the relative displacements of the edge of the hole. Referring to Fig. N.1.2, the constraints are:

$$u_x^* = \pm\ 0.341 \text{ in. for node } 6,$$

$$u_y^* = \pm\ 0.341 \text{ in. for node } 32.$$

The initial and minimum allowable values of the design variables are

$t = 0.055$ in. (thickness of plate),

$A = 0.2145$ in^2 (cross-sectional area of the reinforcing ring),

$t^* = 0.01$ in.,

$A^* = 0.01$ in^2.

The history of the design is summarized in Tables N.1.1 and N.1.2. The problem was run for seven redesign cycles with a relaxation factor of $\alpha = 0.4$. The seventh design has not yet reached the convergence criteria, but it was considered to be sufficiently close to the final design to make further runs unnecessary: the weight change in the last design cycle is small (Table N.1.2), and the optimality indicies of all the design variables are within the acceptable bound, with the exception of design variable No. 5 (see Evaluation of Design No. 7).

Tables N.1.1 and N.1.2 also contain the design obtained by a different optimization technique [11], but regretfully only an order-of-magnitude comparison of the two designs can be made. The problem is that Ref. [11] employed triangular membrane elements, and

allowed the plate thickness to vary linearly within each element. A linear variation of the cross-sectional area was also used for the reinforcement. We attempted to compensate for this discrepancy in modelling by laying out our finite element net such that the center of each element coincides approximately with a nodal point of Ref. [11]. The design variables at these nodal points have been listed in Tables N.1.1 and N.1.2 as the results of Ref. [11].

The comparison is made more difficult by lack of other information in Ref. [11], such as the Poisson's ratio used, and an explanation of how the structural weight was computed (we were unable to duplicate the nondimensional weight listed in Ref. [11]).

The main discrepancy is in the thickness of the membrane elements adjacent to the hole, where our design is considerably heavier. The source of these differences is likely due to the finer finite element mesh that we used around the hole, thereby obtaining a more accurate prediction of the stress concentration.

A noteworthy feature of the design is the small weight increase between designs Nos. 3 and 4 (Table N.1.2). Such weight increments are commonly caused by a change in the active constraints; in this case the appearance of a stress constraint that was inactive in the first three redesigns.

<u>Special notes on input-output</u>:

1) The Von Mises yield criterion only was used in the stress-constrained design of membrane elements. The maximum shear stress theory of failure was made inoperative by specifying $\tau^* = 0$ on the Material Property Cards.

2) The loading acting on the sides of elements 25, 32 and 36 is defined in the Element Data as uniform compression of 25,000 lb/in. The use of the Element Load Multiplier - 0.01 converts this load to the desired tension of 250 lb/in.

3) The use of the stress printout code NS = 3 in Element Data means that the stresses are evaluated at the center only for each membrane element. Stress-constrained redesign is, therefore, based solely on these central stresses.

4) The incompatible displacement modes were not suppressed, since NPAR(6) was left blank on the Element Control Card (see Echo of Input Cards).

Design Variable	Element	Critical, Scaled Designs (thickness in inches)								Ref. [11]
		0	1	2	3	4	5	6	7	
1	1	.1056	.0398	.0100	.0100	.0111	.0104	.0100	.0100	.0100
2	2	.1056	.0565	.0178	.0100	.0111	.0104	.0100	.0100	.0100
3	3	.1056	.0945	.0513	.0117	.0111	.0104	.0100	.0100	.0100
4	4	.1056	.1565	.1120	.0697	.0317	.0104	.0100	.0100	.0146
5	5	.1056	.2396	.2194	.1898	.1727	.1186	.0773	.0522	.1354
6	6	.1056	.3031	.3517	.3986	.5209	.5404	.5844	.6354	.1536
7	7	.1056	.0464	.0116	.0100	.0111	.0104	.0100	.0100	.0100
8	8	.1056	.0660	.0299	.0100	.0111	.0104	.0100	.0100	.0100
9	9	.1056	.0972	.0670	.0377	.0269	.0179	.0128	.0104	.0100
10	10	.1056	.1390	.1261	.0988	.0968	.0865	.0866	.0868	.0347
11	11	.1056	.1879	.1960	.1777	.1895	.1657	.1514	.1425	?
12	12	.1056	.2205	.2812	.3015	.3825	.3835	.3968	.4187	.1809
13	13	.1056	.0693	.0381	.0213	.0148	.0104	.0100	.0100	.0100
14	14	.1056	.0773	.0512	.0336	.0326	.0277	.0250	.0234	.0100
15	15	.1056	.0953	.0820	.0618	.0673	.0632	.0633	.0657	.0480
16	16	.1056	.1103	.1223	.1067	.1194	.1118	.1108	.1121	.0718
17	17	.1056	.1062	.1587	.1590	.1925	.1817	.1782	.1782	?
18	18	.1056	.1047	.1507	.1471	.1787	.1681	.1650	.1663	.0806
19	19	.1056	.0729	.0496	.0352	.0310	.0265	.0248	.0245	.0172
20	20	.1056	.0736	.0544	.0404	.0391	.0338	.0324	.0378	.0264
21	21	.1056	.0866	.0800	.0646	.0694	.0622	.0593	.0591	.0240
22	22	.1056	.0936	.1025	.0917	.1105	.1022	.0979	.0979	.0782
23	23	.1056	.0605	.0443	.0326	.0400	.0338	.0301	.0287	?
24	24	.1056	.0824	.0796	.0667	.0877	.0790	.0742	.0735	.0418
25	25	.1056	.0491	.0244	.0154	.0136	.0135	.0132	.0130	.0102
26	26	.1056	.0531	.0283	.0162	.0115	.0104	.0100	.0100	.0100
27	27	.1056	.0641	.0432	.0273	.0245	.0199	.0177	.0169	.0100
28	28	.1056	.0688	.0519	.0292	.0330	.0315	.0309	.0315	.0189
29	29	.1056	.0622	.0386	.0170	.0111	.0104	.0100	.0100	.0251
30	30	.1056	.0842	.0926	.0810	.1151	.1237	.1287	.1332	?
31	31	.1056	.1407	.1658	.1605	.2284	.2414	.2387	.2400	.0498
32	32	.1056	.0493	.0212	.0110	.0117	.0122	.0212	.0122	.0102
33	33	.1056	.0498	.0235	.0121	.0120	.0110	.0106	.0106	.0100
34	34	.1056	.0437	.0100	.0100	.0111	.0104	.0100	.0100	.0100
35	35	.1056	.0353	.0100	.0100	.0111	.0104	.0100	.0100	.0100
36	36	.1056	.0523	.0208	.0103	.0111	.0104	.0100	.0100	.0100
37	37	.1056	.0397	.0100	.0100	.0111	.0104	.0100	.0100	.0100

Table N.1.1

Design History of Membrane Elements.

Design Variable	Element	Critical, Scaled Designs (areas in sq. in.)								Ref. [11]
		0	1	2	3	4	5	6	7	
38	1	.0215	.1316	.1595	.1880	.2511	.2601	.2718	.2795	.2149
39	2	.0215	.1149	.1298	.1417	.1754	.1684	.1634	.1590	.1767
40	3	.0215	.0833	.1018	.1039	.1233	.1148	.1088	.1047	.0768
41	4	.0215	.0315	.0375	.0332	.0372	.0331	.0356	.0289	.0207
42	5	.0215	.0202	.0100	.0100	.0111	.0104	.0100	.0100	.0008
43	6	.0215	.0313	.0223	.0137	.0111	.0104	.0100	.0100	.0256
Wt. (lb)		.8656	.5672	.3992	.2959	.3121	.2862	.2763	.2759	?

Table N.1.2

Design History of Bar Elements and Total Structural Weight

```
00001 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
05400  RECTANGULAR PLATE WITH REINFORCED HOLE - STRESS AND DISPLACEMENT CONSTS.
05450    42    2    1   43
05500    10         1 0.025     0.1          1    1    1 0.8        0.4
05550    37 0.055     0.01
05600    43 0.02145   0.01
05650     1         1   -1   -1   -1   -1
05700     2         1                     0.5
05750     3         1                     1.7
05800     4         1                     2.7
05850     5         1                     3.5
05900     6         1                     3.9
05950     7                                          0.8
06000     8                               0.5        0.8
06050     9                                1.8      0.8
06100    10                                2.8      0.75
06150    11                               3.6       0.7
06200    12                                3.91      0.34
06250    13                                3.98     0.78
06300    14                                          2.7
06350    15                                0.7      2.6
06400    16                               1.9       2.5
06450    17                                3.        2.4
06500    18                               3.8        1.7
06550    19                                4.36      1.83
06600    20                                         5.3
06650    21                                0.8      5.
06700    22                                2.5       4.3
06750    23                                3.7       3.7
06800    24                                4.7       3.2
06850    25                               5.56       3.19
06900    26                                          8.1
06950    27                                1.7       7.3
07000    28                               3.6        6.1
07050    29                                4.7       5.2
07100    30                                6.1      4.2
07150    31                                7.02      3.82
07200    32    1                          7.8        3.9
07250    33                                          11.7
07300    34                                1.8       11.7
07350    35                                3.2      9.4
07400    36                                4.8       7.7
07450    37                                6.2       6.3
07500    38    1                           7.8       4.9
07550    39                                4.9       11.7
07600    40                                6.2       9.3
07650    41    1                           7.8       7.4
07700    42    1         1    1    1    1  7.8       11.7
07750     1    6    1    1    1    1
07800     1    1  0.1
07850            1000000.            25000.     25000.
07900     1      2.145
07950
08000
08050
08100
08150     1    6   12    1    1   38
08200     2   12   13    1    1   39
08250     3   13   19    1    1   40
08251 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
10950
```

Echo of Input Cards

```
08252 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
08300     4   19   25    1    1   41
08350     5   25   31    1    1   42
08400     6   31   32    1    1   43
08450     3   37    1    1    2
08500     1    1  0.1
08550           1000000.  0.25                25000.     25000.
08600            -0.01
08650
08700
08750
08800     1    1    2    8    7    1    1
08850     2    2    3    9    8    1    2
08900     3    3    4   10    9    1    3
08950     4    4    5   11   10    1    4
09000     5    5    6   12   11    1    5
09050     6   11   12   13   13    1    6
09100     7    7    8   15   14    1    7
09150     8    8    9   16   15    1    8
09200     9    9   10   17   16    1    9
09250    10   10   11   18   17    1   10
09300    11   11   13   18   18    1   11
09350    12   13   19   18   18    1   12
09400    13   14   15   21   20    1   13
09450    14   15   16   22   21    1   14
09500    15   16   17   23   22    1   15
09550    16   17   18   24   23    1   16
09600    17   18   19   24   24    1   17
09650    18   19   25   24   24    1   18
09700    19   20   21   27   26    1   19
09750    20   21   22   28   27    1   20
09800    21   22   23   29   28    1   21
09850    22   23   24   30   29    1   22
09900    23   24   25   30   30    1   23
09950    24   25   31   30   30    1   24
10000    25   34   33   26   26    1   25                     25000.
10050    26   26   27   35   34    1   26
10100    27   27   28   36   35    1   27
10150    28   28   29   37   36    1   28
10200    29   29   30   38   37    1   29
10250    30   30   31   38   38    1   30
10300    31   31   32   38   38    1   31
10350    32   39   34   35   35    1   32                     25000.
10400    33   35   36   40   39    1   33
10450    34   36   37   41   40    1   34
10500    35   37   38   41   41    1   35
10550    36   42   39   40   40    1   36                     25000.
10600    37   40   41   42   42    1   37
10650    1.0
10700     6  1.0341                        -.0341
10750    32  1     .0341                          -.0341
10800
10850
10900 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
10950
```

RECTANGULAR PLATE WITH REINFORCED HOLE - STRESS AND DISPLACEMENT CONSTS.

```
NUMBER OF NODAL POINTS   =  42
NUMBER OF ELEMENT TYPES  =   2
NUMBER OF LOAD CASES     =   1
NUMBER OF DES. VARIABLES =  43
```

DESIGN CONTROL DATA

```
NCYCL =    10
KSCALE=     1
DELTA =  0.2500E-01
EPSIL =  0.1000E 00
KDISP =     1
OMEGA =   0.80000
ALPA  =   0.40000
```

DESIGN VARIABLE INPUT DATA

DESIGN VARIABLE NUMBER	INITIAL VALUE	MIN ALLOWABLE VALUE
1	0.5500E-01	0.1000E-01
2	0.5500E-01	0.1000E-01
3	0.5500E-01	0.1000E-01
4	0.5500E-01	0.1000E-01
5	0.5500E-01	0.1000E-01
6	0.5500E-01	0.1000E-01
7	0.5500E-01	0.1000E-01
8	0.5500E-01	0.1000E-01
9	0.5500E-01	0.1000E-01
10	0.5500E-01	0.1000E-01
11	0.5500E-01	0.1000E-01
12	0.5500E-01	0.1000E-01
13	0.5500E-01	0.1000E-01
14	0.5500E-01	0.1000E-01
15	0.5500E-01	0.1000E-01
16	0.5500E-01	0.1000E-01
17	0.5500E-01	0.1000E-01
18	0.5500E-01	0.1000E-01
19	0.5500E-01	0.1000E-01
20	0.5500E-01	0.1000E-01
21	0.5500E-01	0.1000E-01
22	0.5500E-01	0.1000E-01
23	0.5500E-01	0.1000E-01
24	0.5500E-01	0.1000E-01
25	0.5500E-01	0.1000E-01
26	0.5500E-01	0.1000E-01
27	0.5500E-01	0.1000E-01
28	0.5500E-01	0.1000E-01
29	0.5500E-01	0.1000E-01
30	0.5500E-01	0.1000E-01
31	0.5500E-01	0.1000E-01
32	0.5500E-01	0.1000E-01
33	0.5500E-01	0.1000E-01
34	0.5500E-01	0.1000E-01
35	0.5500E-01	0.1000E-01

Computer Printout

(Input data, the initial design and the final design only are reproduced.)

36	0.5500E-01	0.1000E-01
37	0.5500E-01	0.1000E-01
38	0.2145E-01	0.1000E-01
39	0.2145E-01	0.1000E-01
40	0.2145E-01	0.1000E-01
41	0.2145E-01	0.1000E-01
42	0.2145E-01	0.1000E-01
43	0.2145E-01	0.1000E-01

NODAL POINT INPUT DATA

NODE	BOUNDARY CONDITION CODES						/------NODAL POINT COORDINATES--------/				
NUMBER	X	Y	Z	XX	YY	ZZ	X	Y	Z		T
1	0	1	-1	-1	-1	-1	0.0	0.0	0.0	0	0.0
2	0	1	0	0	0	0	0.500	0.0	0.0	0	0.0
3	0	1	0	0	0	0	1.700	0.0	0.0	0	0.0
4	0	1	0	0	0	0	2.700	0.0	0.0	0	0.0
5	0	1	0	0	0	0	3.500	0.0	0.0	0	0.0
6	0	1	0	0	0	0	3.900	0.0	0.0	0	0.0
7	0	0	0	0	0	0	0.0	0.800	0.0	0	0.0
8	0	0	0	0	0	0	0.500	0.800	0.0	0	0.0
9	0	0	0	0	0	0	1.800	0.800	0.0	0	0.0
10	0	0	0	0	0	0	2.800	0.750	0.0	0	0.0
11	0	0	0	0	0	0	3.600	0.700	0.0	0	0.0
12	0	0	0	0	0	0	3.910	0.340	0.0	0	0.0
13	0	0	0	0	0	0	3.980	0.780	0.0	0	0.0
14	0	0	0	0	0	0	0.0	2.700	0.0	0	0.0
15	0	0	0	0	0	0	0.700	2.600	0.0	0	0.0
16	0	0	0	0	0	0	1.900	2.500	0.0	0	0.0
17	0	0	0	0	0	0	3.000	2.400	0.0	0	0.0
18	0	0	0	0	0	0	3.800	1.700	0.0	0	0.0
19	0	0	0	0	0	0	4.360	1.830	0.0	0	0.0
20	0	0	0	0	0	0	0.0	5.300	0.0	0	0.0
21	0	0	0	0	0	0	0.800	5.000	0.0	0	0.0
22	0	0	0	0	0	0	2.500	4.300	0.0	0	0.0
23	0	0	0	0	0	0	3.700	3.700	0.0	0	0.0
24	0	0	0	0	0	0	4.700	3.200	0.0	0	0.0
25	0	0	0	0	0	0	5.560	3.190	0.0	0	0.0
26	0	0	0	0	0	0	0.0	8.100	0.0	0	0.0
27	0	0	0	0	0	0	1.700	7.300	0.0	0	0.0
28	0	0	0	0	0	0	3.600	6.100	0.0	0	0.0
29	0	0	0	0	0	0	4.700	5.200	0.0	0	0.0
30	0	0	0	0	0	0	6.100	4.200	0.0	0	0.0
31	0	0	0	0	0	0	7.020	3.820	0.0	0	0.0
32	1	0	0	0	0	0	7.800	3.900	0.0	0	0.0
33	0	0	0	0	0	0	0.0	11.700	0.0	0	0.0
34	0	0	0	0	0	0	1.800	11.700	0.0	0	0.0
35	0	0	0	0	0	0	3.200	9.400	0.0	0	0.0
36	0	0	0	0	0	0	4.800	7.700	0.0	0	0.0
37	0	0	0	0	0	0	6.200	6.300	0.0	0	0.0
38	1	0	0	0	0	0	7.800	4.900	0.0	0	0.0
39	0	0	0	0	0	0	4.800	11.700	0.0	0	0.0
40	0	0	0	0	0	0	6.200	9.300	0.0	0	0.0
41	1	0	0	0	0	0	7.800	7.400	0.0	0	0.0
42	1	0	1	1	1	1	7.800	11.700	0.0	0	0.0

GENERATED NODAL DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	/------NODAL POINT COORDINATES--------/ X	Y	Z	T
1	0	1	-1	-1	-1	-1	0.0	0.0	0.0	0.0
2	0	1	-1	-1	-1	-1	0.500	0.0	0.0	0.0
3	0	1	-1	-1	-1	-1	1.700	0.0	0.0	0.0
4	0	1	-1	-1	-1	-1	2.700	0.0	0.0	0.0
5	0	1	-1	-1	-1	-1	3.500	0.0	0.0	0.0
6	0	1	-1	-1	-1	-1	3.900	0.0	0.0	0.0
7	0	0	-1	-1	-1	-1	0.0	0.800	0.0	0.0
8	0	0	-1	-1	-1	-1	0.500	0.800	0.0	0.0
9	0	0	-1	-1	-1	-1	1.800	0.800	0.0	0.0
10	0	0	-1	-1	-1	-1	2.800	0.750	0.0	0.0
11	0	0	-1	-1	-1	-1	3.600	0.700	0.0	0.0
12	0	0	-1	-1	-1	-1	3.910	0.340	0.0	0.0
13	0	0	-1	-1	-1	-1	3.980	0.780	0.0	0.0
14	0	0	-1	-1	-1	-1	0.0	2.700	0.0	0.0
15	0	0	-1	-1	-1	-1	0.700	2.600	0.0	0.0
16	0	0	-1	-1	-1	-1	1.900	2.500	0.0	0.0
17	0	0	-1	-1	-1	-1	3.000	2.400	0.0	0.0
18	0	0	-1	-1	-1	-1	3.800	1.700	0.0	0.0
19	0	0	-1	-1	-1	-1	4.360	1.830	0.0	0.0
20	0	0	-1	-1	-1	-1	0.0	5.300	0.0	0.0
21	0	0	-1	-1	-1	-1	0.800	5.000	0.0	0.0
22	0	0	-1	-1	-1	-1	2.500	4.300	0.0	0.0
23	0	0	-1	-1	-1	-1	3.700	3.700	0.0	0.0
24	0	0	-1	-1	-1	-1	4.700	3.200	0.0	0.0
25	0	0	-1	-1	-1	-1	5.560	3.190	0.0	0.0
26	0	0	-1	-1	-1	-1	0.0	8.100	0.0	0.0
27	0	0	-1	-1	-1	-1	1.700	7.300	0.0	0.0
28	0	0	-1	-1	-1	-1	3.600	6.100	0.0	0.0
29	0	0	-1	-1	-1	-1	4.700	5.200	0.0	0.0
30	0	0	-1	-1	-1	-1	6.100	4.200	0.0	0.0
31	0	0	-1	-1	-1	-1	7.020	3.820	0.0	0.0
32	1	0	-1	-1	-1	-1	7.800	3.900	0.0	0.0
33	0	0	-1	-1	-1	-1	0.0	11.700	0.0	0.0
34	0	0	-1	-1	-1	-1	1.800	11.700	0.0	0.0
35	0	0	-1	-1	-1	-1	3.200	9.400	0.0	0.0
36	0	0	-1	-1	-1	-1	4.800	7.700	0.0	0.0
37	0	0	-1	-1	-1	-1	6.200	6.300	0.0	0.0
38	1	0	-1	-1	-1	-1	7.800	4.900	0.0	0.0
39	0	0	-1	-1	-1	-1	4.800	11.700	0.0	0.0
40	0	0	-1	-1	-1	-1	6.200	9.300	0.0	0.0
41	1	0	-1	-1	-1	-1	7.800	7.400	0.0	0.0
42	1	0	1	1	1	1	7.800	11.700	0.0	0.0

EQUATION NUMBERS

N	X	Y	Z	XX	YY	ZZ
1	1	0	0	0	0	0
2	2	0	0	0	0	0
3	3	0	0	0	0	0
4	4	0	0	0	0	0
5	5	0	0	0	0	0
6	6	0	0	0	0	0
7	7	8	0	0	0	0
8	9	10	0	0	0	0
9	11	12	0	0	0	0
10	13	14	0	0	0	0

11	15	16	0	0	0	0
12	17	18	0	0	0	0
13	19	20	0	0	0	0
14	21	22	0	0	0	0
15	23	24	0	0	0	0
16	25	26	0	0	0	0
17	27	28	0	0	0	0
18	29	30	0	0	0	0
19	31	32	0	0	0	0
20	33	34	0	0	0	0
21	35	36	0	0	0	0
22	37	38	0	0	0	0
23	39	40	0	0	0	0
24	41	42	0	0	0	0
25	43	44	0	0	0	0
26	45	46	0	0	0	0
27	47	48	0	0	0	0
28	49	50	0	0	0	0
29	51	52	0	0	0	0
30	53	54	0	0	0	0
31	55	56	0	0	0	0
32	0	57	0	0	0	0
33	58	59	0	0	0	0
34	60	61	0	0	0	0
35	62	63	0	0	0	0
36	64	65	0	0	0	0
37	66	67	0	0	0	0
38	0	68	0	0	0	0
39	69	70	0	0	0	0
40	71	72	0	0	0	0
41	0	73	0	0	0	0
42	0	74	0	0	0	0

```
NUMBER OF TRUSS ELEMENTS                  =    6
CONSTRUCTION CODE                         =    1
NUMBER OF MATERIALS                       =    1
NUMBER OF TEMPS FOR WHICH MATL PROPS GIVEN=    1
NUMBER OF DIFFERENT GEOMETRIES PROPS GIVEN=    1
```

MATERIAL PROPERTY CARDS

MATERIAL NUMBER	NUMBER OF TEMPS	SPECIFIC WEIGHT	TEMP	YOUNGS MODULUS	COEFFT OF THERM EXPAN	/--ALLOWABLE STRESSES--/ TENSION	COMPRESSION
1	1	0.1000E 00	0.0	0.1000E 07	0.0	0.2500E 05	0.2500E 05

GEOMETRIC PROPERTY CARDS

GEOMETRY NUMBER	X-SECT AREA	/--MOMENTS OF INERTIA--/ YY	ZZ
1	0.2145D 01	0.1000E 07	0.1000E 07

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.0	0.0	0.0	0.0
Y-DIR	0.0	0.0	0.0	0.0
Z-DIR	0.0	0.0	0.0	0.0
TEMP	0.0	0.0	0.0	0.0

PROCESSED ELEMENT DATA

ELEMENT NUMBER	/-NODE NOS-/ I	J	/--ELEMENT ID NOS-/ MATL	GEOMY	D VAR	DESIGN VAR FRACTION	REFERENCE TEMP	END FIXITY COEFFICIENTS YY	ZZ	BAND WIDTH
1	6	12	1	1	38	0.1000E 01	0.0	0.1000D 01	0.1000D 01	13
2	12	13	1	1	39	0.1000E 01	0.0	0.1000D 01	0.1000D 01	4
3	13	19	1	1	40	0.1000E 01	0.0	0.1000D 01	0.1000D 01	14
4	19	25	1	1	41	0.1000E 01	0.0	0.1000D 01	0.1000D 01	14
5	25	31	1	1	42	0.1000E 01	0.0	0.1000D 01	0.1000D 01	14
6	31	32	1	1	43	0.1000E 01	0.0	0.1000D 01	0.1000D 01	3

NUMBER OF MEMBRANE ELEMENTS = 37
CONSTRUCTION KODE = 2
NUMBER OF MATERIALS = 1
NUMBER OF TEMPS FOR WHICH MATL PROPS GIVEN= 1

MATERIAL PROPERTY CARDS

MATL NBR	NO OF TEMP	SPECIFIC WEIGHT	TEMPERATURE	YOUNGS MODULUS	POISSONS RATIO	COEFFT OF THERM EXPN	/-----------ALLOWABLE STRESSES-----------/ TENSION	COMPRESSION	SHEAR
1	1	0.1000E 00	0.0	0.1000E 07	0.2500E 00	0.0	0.2500E 05	0.2500E 05	0.0

ELEMENT LOAD FRACTIONS

LOAD CASE	TEMPERATURE	PRESSURE	X-DIRECTION	Y-DIRECTION	Z-DIRECTION
A	0.0	-0.010	0.0	0.0	0.0
B	0.0	0.0	0.0	0.0	0.0
C	0.0	0.0	0.0	0.0	0.0
D	0.0	0.0	0.0	0.0	0.0

PROCESSED ELEMENT DATA

ELEMT/ NUMBR	-------NODES------/ I	J	K	L	/-ID NOS-/ MAT	DV	DES VAR FRACTION	REFERENCE TEMP	PRESSURE	BETA	PRNT CODE	BAND WDTH
1	1	2	8	7	1	1	0.1000E 01	0.0	0.0	0.0	3	10
2	2	3	9	8	1	2	0.1000E 01	0.0	0.0	0.0	3	11
3	3	4	10	9	1	3	0.1000E 01	0.0	0.0	0.0	3	12
4	4	5	11	10	1	4	0.1000E 01	0.0	0.0	0.0	3	13
5	5	6	12	11	1	5	0.1000E 01	0.0	0.0	0.0	3	14
6	11	12	13	13	1	6	0.1000E 01	0.0	0.0	0.0	3	6
7	7	8	15	14	1	7	0.1000E 01	0.0	0.0	0.0	3	18
8	8	9	16	15	1	8	0.1000E 01	0.0	0.0	0.0	3	18
9	9	10	17	16	1	9	0.1000E 01	0.0	0.0	0.0	3	18
10	10	11	18	17	1	10	0.1000E 01	0.0	0.0	0.0	3	18
11	11	13	18	18	1	11	0.1000E 01	0.0	0.0	0.0	3	16
12	13	19	18	18	1	12	0.1000E 01	0.0	0.0	0.0	3	14
13	14	15	21	20	1	13	0.1000E 01	0.0	0.0	0.0	3	16
14	15	16	22	21	1	14	0.1000E 01	0.0	0.0	0.0	3	16
15	16	17	23	22	1	15	0.1000E 01	0.0	0.0	0.0	3	16
16	17	18	24	23	1	16	0.1000E 01	0.0	0.0	0.0	3	16
17	18	19	24	24	1	17	0.1000E 01	0.0	0.0	0.0	3	14
18	19	25	24	24	1	18	0.1000E 01	0.0	0.0	0.0	3	14
19	20	21	27	26	1	19	0.1000E 01	0.0	0.0	0.0	3	16
20	21	22	28	27	1	20	0.1000E 01	0.0	0.0	0.0	3	16
21	22	23	29	28	1	21	0.1000E 01	0.0	0.0	0.0	3	16
22	23	24	30	29	1	22	0.1000E 01	0.0	0.0	0.0	3	16
23	24	25	30	30	1	23	0.1000E 01	0.0	0.0	0.0	3	14
24	25	31	30	30	1	24	0.1000E 01	0.0	0.0	0.0	3	14
25	34	33	26	26	1	25	0.1000E 01	0.0	0.2500D 05	0.0	3	17
26	26	27	35	34	1	26	0.1000E 01	0.0	0.0	0.0	3	19
27	27	28	36	35	1	27	0.1000E 01	0.0	0.0	0.0	3	19
28	28	29	37	36	1	28	0.1000E 01	0.0	0.0	0.0	3	19
29	29	30	38	37	1	29	0.1000E 01	0.0	0.0	0.0	3	18
30	30	31	38	38	1	30	0.1000E 01	0.0	0.0	0.0	3	16
31	31	32	38	38	1	31	0.1000E 01	0.0	0.0	0.0	3	14
32	39	34	35	35	1	32	0.1000E 01	0.0	0.2500D 05	0.0	3	11
33	35	36	40	39	1	33	0.1000E 01	0.0	0.0	0.0	3	11

34	36	37	41	40	1	34	0.1000E 01	0.0	0.0	0.0	3	10
35	37	38	41	41	1	35	0.1000E 01	0.0	0.0	0.0	3	8
36	42	39	40	40	1	36	0.1000E 01	0.0	0.2500D 05	0.0	3	6
37	40	41	42	42	1	37	0.1000E 01	0.0	0.0	0.0	3	4

STRUCTURE LOAD CASE	STRUCTURE LOAD MULTIPLIERS A	B	C	D
1	1.000	0.0	0.0	0.0

NODAL DISPLACEMENT/ROTATION CONSTRAINTS

NODE NO.	LOAD/ CASE	DX	DY	DZ	RX	RY	RZ	-DX	-DY	-DZ	-RX	-RY	-RZ
		-----	-----	-----	----MAX.ALLOWABLE	DISPLACEMENTS	AND ROTATIONS----	-----	-----	-----	-----	-----	-----/
6	1	0.03410	0.0	0.0	0.0	0.0	0.0	-0.03410	0.0	0.0	0.0	0.0	0.0
32	1	0.0	0.03410	0.0	0.0	0.0	0.0	0.0	-0.03410	0.0	0.0	0.0	0.0

NODAL POINT LOADS

NODE NO.	LOAD CASE	RX	RY	RZ	MX	MY	MZ
			APPLIED LOADS				

TOTAL NUMBER OF EQUATIONS = 74
BANDWIDTH = 19
NUMBER OF EQUATIONS IN A BLOCK = 47
NUMBER OF BLOCKS = 2

****
ANALYSIS OF DESIGN NUMBER 0
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
42	1	0.0	8.646E-02	0.0	0.0	0.0	0.0
41	1	0.0	7.082E-02	0.0	0.0	0.0	0.0
40	1	-2.878E-04	7.563E-02	0.0	0.0	0.0	0.0
39	1	-4.915E-03	8.134E-02	0.0	0.0	0.0	0.0
38	1	0.0	6.712E-02	0.0	0.0	0.0	0.0
37	1	1.088E-03	6.581E-02	0.0	0.0	0.0	0.0
36	1	1.213E-03	6.412E-02	0.0	0.0	0.0	0.0
35	1	6.326E-04	6.516E-02	0.0	0.0	0.0	0.0
34	1	-4.914E-03	6.986E-02	0.0	0.0	0.0	0.0
33	1	-2.869E-03	6.421E-02	0.0	0.0	0.0	0.0
32	1	0.0	6.548E-02	0.0	0.0	0.0	0.0
31	1	4.052E-03	6.414E-02	0.0	0.0	0.0	0.0
30	1	5.430E-03	5.966E-02	0.0	0.0	0.0	0.0
29	1	5.867E-03	5.249E-02	0.0	0.0	0.0	0.0
28	1	5.737E-03	5.024E-02	0.0	0.0	0.0	0.0
27	1	5.960E-03	4.721E-02	0.0	0.0	0.0	0.0
26	1	7.258E-03	4.696E-02	0.0	0.0	0.0	0.0
25	1	1.417E-02	5.178E-02	0.0	0.0	0.0	0.0
24	1	1.600E-02	4.181E-02	0.0	0.0	0.0	0.0
23	1	1.559E-02	3.580E-02	0.0	0.0	0.0	0.0
22	1	1.587E-02	3.258E-02	0.0	0.0	0.0	0.0
21	1	1.595E-02	2.901E-02	0.0	0.0	0.0	0.0
20	1	1.552E-02	2.666E-02	0.0	0.0	0.0	0.0
19	1	2.812E-02	2.849E-02	0.0	0.0	0.0	0.0
18	1	3.001E-02	2.092E-02	0.0	0.0	0.0	0.0
17	1	2.657E-02	2.108E-02	0.0	0.0	0.0	0.0

16	1	2.837E-02	1.701E-02	0.0	0.0	0.0	0.0
15	1	2.964E-02	1.280E-02	0.0	0.0	0.0	0.0
14	1	3.017E-02	1.017E-02	0.0	0.0	0.0	0.0
13	1	3.497E-02	1.224E-02	0.0	0.0	0.0	0.0
12	1	3.722E-02	5.331E-03	0.0	0.0	0.0	0.0
11	1	3.597E-02	8.326E-03	0.0	0.0	0.0	0.0
10	1	3.678E-02	6.509E-03	0.0	0.0	0.0	0.0
9	1	3.712E-02	5.166E-03	0.0	0.0	0.0	0.0
8	1	3.843E-02	3.305E-03	0.0	0.0	0.0	0.0
7	1	3.884E-02	1.875E-03	0.0	0.0	0.0	0.0
6	1	3.675E-02	0.0	0.0	0.0	0.0	0.0
5	1	3.785E-02	0.0	0.0	0.0	0.0	0.0
4	1	3.801E-02	0.0	0.0	0.0	0.0	0.0
3	1	3.868E-02	0.0	0.0	0.0	0.0	0.0
2	1	3.942E-02	0.0	0.0	0.0	0.0	0.0
1	1	3.985E-02	0.0	0.0	0.0	0.0	0.0

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01
10	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01
20	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01
30	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.5500E-01	0.2145E-01	0.2145E-01	0.2145E-01
40	0.2145E-01	0.2145E-01	0.2145E-01							

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT	X-SECT AREA	LOAD COND	AXIAL FORCE
1	0.2145E-01	1	0.3369E 03
2	0.2145E-01	1	0.3113E 03
3	0.2145E-01	1	0.2488E 03
4	0.2145E-01	1	0.9733E 02
5	0.2145E-01	1	-0.5920E 02
6	0.2145E-01	1	-0.1065E 03

ANALYSIS OF MEMBRANE ELEMENTS, CONSTRN CODE= 2

ELEMENT	SHEET THICKNESS	LOAD COND	LOCATION	/---MEMBRANE FORCES IN LOCAL COORDS---/ NXX	NYY	NXY	/--MEMBRANE FORCES IN MATERIAL COORDS-/ N11	N22	N12
1	0.5500E-01	1	CEN	-0.1910E 01	0.1776E 03	0.3921E 01	-0.1910E 01	0.1776E 03	0.3921E 01
2	0.5500E-01	1	CEN	0.2901E 02	0.2959E 03	-0.1749E 02	0.2901E 02	0.2959E 03	-0.1749E 02
3	0.5500E-01	1	CEN	0.7665E 02	0.4273E 03	-0.1916E 02	0.7665E 02	0.4273E 03	-0.1916E 02
4	0.5500E-01	1	CEN	0.1074E 03	0.5786E 03	-0.1324E 02	0.1074E 03	0.5786E 03	-0.1324E 02
5	0.5500E-01	1	CEN	0.1634E 03	0.7496E 03	0.2219E 02	0.1634E 03	0.7496E 03	0.2219E 02
6	0.5500E-01	1	CEN	0.4761E 03	0.4737E 03	-0.3588E 03	0.4761E 03	0.4737E 03	-0.3588E 03
7	0.5500E-01	1	CEN	-0.6145E 00	0.2532E 03	-0.1955E 02	-0.6145E 00	0.2532E 03	-0.1955E 02
8	0.5500E-01	1	CEN	0.1403E 02	0.3263E 03	-0.4935E 02	0.1403E 02	0.3263E 03	-0.4935E 02
9	0.5500E-01	1	CEN	0.3486E 02	0.4193E 03	-0.7257E 02	0.3486E 02	0.4193E 03	-0.7257E 02
10	0.5500E-01	1	CEN	0.7451E 02	0.5334E 03	-0.3932E 02	0.7451E 02	0.5334E 03	-0.3932E 02
11	0.5500E-01	1	CEN	0.1216E 03	0.5809E 03	0.1565E 03	0.1216E 03	0.5809E 03	0.1565E 03
12	0.5500E-01	1	CEN	0.6502E 03	0.4867E 02	0.1061E 03	0.6502E 03	0.4867E 02	0.1061E 03
13	0.5500E-01	1	CEN	0.1320E 02	0.3439E 03	-0.6197E 02	0.1320E 02	0.3439E 03	-0.6197E 02
14	0.5500E-01	1	CEN	-0.1527E 02	0.3611E 03	-0.6122E 02	-0.1527E 02	0.3611E 03	-0.6122E 02
15	0.5500E-01	1	CEN	-0.4969E 02	0.3900E 03	-0.6121E 02	-0.4969E 02	0.3900E 03	-0.6121E 02
16	0.5500E-01	1	CEN	0.1312E 03	0.2431E 03	-0.2283E 03	0.1312E 03	0.2431E 03	-0.2283E 03
17	0.5500E-01	1	CEN	0.6864E 02	0.3238E 03	0.1473E 03	0.6864E 02	0.3238E 03	0.1473E 03
18	0.5500E-01	1	CEN	0.2670E 03	0.6971E 02	0.1884E 03	0.2670E 03	0.6971E 02	0.1884E 03
19	0.5500E-01	1	CEN	0.3097E 02	0.3394E 03	-0.1117E 03	0.3097E 02	0.3394E 03	-0.1117E 03
20	0.5500E-01	1	CEN	0.7399E 00	0.3012E 03	-0.1100E 03	0.7399E 00	0.3012E 03	-0.1100E 03
21	0.5500E-01	1	CEN	-0.3272E 02	0.2869E 03	-0.1305E 03	-0.3272E 02	0.2869E 03	-0.1305E 03
22	0.5500E-01	1	CEN	-0.9936E 02	0.2579E 03	-0.1003E 03	-0.9936E 02	0.2579E 03	-0.1003E 03
23	0.5500E-01	1	CEN	-0.1090E 03	0.6321E 02	0.8931E 02	-0.1090E 03	0.6321E 02	0.8931E 02
24	0.5500E-01	1	CEN	-0.1199E 03	0.1277E 03	0.1377E 03	-0.1199E 03	0.1277E 03	0.1377E 03
25	0.5500E-01	1	CEN	0.3609E 01	0.2644E 03	0.7217E 01	0.3609E 01	0.2644E 03	0.7217E 01
26	0.5500E-01	1	CEN	0.4011E 02	0.2525E 03	-0.8755E 02	0.4011E 02	0.2525E 03	-0.8755E 02
27	0.5500E-01	1	CEN	0.1701E 02	0.2612E 03	-0.9821E 02	0.1701E 02	0.2612E 03	-0.9821E 02
28	0.5500E-01	1	CEN	-0.7756E 01	0.2115E 03	-0.1143E 03	-0.7756E 01	0.2115E 03	-0.1143E 03
29	0.5500E-01	1	CEN	-0.7436E 02	0.1191E 03	-0.7791E 02	-0.7436E 02	0.1191E 03	-0.7791E 02
30	0.5500E-01	1	CEN	-0.1750E 03	-0.4039E 02	0.1278E 02	-0.1750E 03	-0.4039E 02	0.1278E 02
31	0.5500E-01	1	CEN	-0.2708E 03	0.9388E 01	0.6394E 02	-0.2708E 03	0.9388E 01	0.6394E 02
32	0.5500E-01	1	CEN	0.6411E 02	0.2565E 03	0.3108E 02	0.6411E 02	0.2565E 03	0.3108E 02
33	0.5500E-01	1	CEN	0.1048E 03	0.1846E 03	-0.1052E 03	0.1048E 03	0.1846E 03	-0.1052E 03
34	0.5500E-01	1	CEN	0.5724E 02	0.1496E 03	-0.8187E 02	0.5724E 02	0.1496E 03	-0.8187E 02
35	0.5500E-01	1	CEN	-0.2292E 02	0.8165E 02	-0.4095E 02	-0.2292E 02	0.8165E 02	-0.4095E 02
36	0.5500E-01	1	CEN	0.1456E 03	0.2220E 03	0.1615E 02	0.1456E 03	0.2220E 03	0.1615E 02
37	0.5500E-01	1	CEN	0.1245E 03	0.1555E 03	-0.7986E 02	0.1245E 03	0.1555E 03	-0.7986E 02

****
EVALUATION OF DESIGN NUMBER 0
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.6282E 00	1	38
MIN	0.1818E 00	0	1

MAX DISP RATIOS	LOAD COND	EQN NUMBER
0.1920E 01	1	57
0.1078E 01	1	6

UNIFORM SCALING OPERATION FOLLOWS

SCALE FACTOR IS 1.920AND DETERMINED BY DISPLACEMENT CONSTRAINTS

DESIGN VARIABLES OF SCALED (CRITICAL) DESIGN ARE

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00
10	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00
20	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00
30	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.1056E 00	0.4119E-01	0.4119E-01	0.4119E-01
40	0.4119E-01	0.4119E-01	0.4119E-01							

STRUCTURAL WEIGHT= 0.8656E 00

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR DISPT. CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	ACT	0.79720E-01
2	ACT	-0.16619E 00
3	ACT	-0.72560E 00
4	ACT	-0.16397E 01
5	ACT	-0.28648E 01
6	ACT	-0.38002E 01
7	ACT	-0.16437E-01
8	ACT	-0.30552E 00
9	ACT	-0.76658E 00
10	ACT	-0.13821E 01
11	ACT	-0.21025E 01
12	ACT	-0.25827E 01
13	ACT	-0.35492E 00
14	ACT	-0.47191E 00
15	ACT	-0.73730E 00
16	ACT	-0.95848E 00
17	ACT	-0.89794E 00
18	ACT	-0.87708E 00
19	ACT	-0.40820E 00
20	ACT	-0.41774E 00
21	ACT	-0.60940E 00
22	ACT	-0.71355E 00
23	ACT	-0.22433E 00
24	ACT	-0.54710E 00
25	ACT	-0.57062E-01
26	ACT	-0.11658E 00
27	ACT	-0.27831E 00
28	ACT	-0.34672E 00
29	ACT	-0.24949E 00
30	ACT	-0.57441E 00
31	ACT	-0.14073E 01
32	ACT	-0.59266E-01
33	ACT	-0.66906E-01

34	ACT	0.22911E-01
35	ACT	0.14580E 00
36	ACT	-0.10436E 00
37	ACT	0.81451E-01
38	ACT	-0.43053E 01
39	ACT	-0.36766E 01
40	ACT	-0.24801E 01
41	ACT	-0.52199E 00
42	ACT	-0.95278E-01
43	ACT	-0.51752E 00

NO. OF ACTIVE DISPLACEMENT CONSTRAINTS ARE 1

****
ANALYSIS OF DESIGN NUMBER 7
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
42	1	0.0	1.720E-01	0.0	0.0	0.0	0.0
41	1	0.0	7.554E-02	0.0	0.0	0.0	0.0
40	1	6.938E-03	1.118E-01	0.0	0.0	0.0	0.0
39	1	1.155E-02	1.589E-01	0.0	0.0	0.0	0.0
38	1	0.0	3.551E-02	0.0	0.0	0.0	0.0
37	1	6.467E-03	5.329E-02	0.0	0.0	0.0	0.0
36	1	1.091E-02	5.856E-02	0.0	0.0	0.0	0.0
35	1	2.521E-02	1.021E-01	0.0	0.0	0.0	0.0
34	1	3.298E-02	1.581E-01	0.0	0.0	0.0	0.0
33	1	4.162E-02	1.580E-01	0.0	0.0	0.0	0.0
32	1	0.0	3.412E-02	0.0	0.0	0.0	0.0
31	1	1.211E-03	3.247E-02	0.0	0.0	0.0	0.0
30	1	9.910E-04	2.859E-02	0.0	0.0	0.0	0.0
29	1	6.769E-03	2.916E-02	0.0	0.0	0.0	0.0
28	1	1.320E-02	3.868E-02	0.0	0.0	0.0	0.0
27	1	2.772E-02	6.079E-02	0.0	0.0	0.0	0.0
26	1	4.175E-02	8.868E-02	0.0	0.0	0.0	0.0
25	1	3.169E-03	2.337E-02	0.0	0.0	0.0	0.0
24	1	3.788E-03	1.914E-02	0.0	0.0	0.0	0.0
23	1	5.680E-03	2.130E-02	0.0	0.0	0.0	0.0
22	1	9.415E-03	2.953E-02	0.0	0.0	0.0	0.0
21	1	1.429E-02	5.180E-02	0.0	0.0	0.0	0.0
20	1	1.607E-02	5.951E-02	0.0	0.0	0.0	0.0
19	1	5.537E-03	1.125E-02	0.0	0.0	0.0	0.0
18	1	6.412E-03	9.095E-03	0.0	0.0	0.0	0.0
17	1	6.275E-03	1.589E-02	0.0	0.0	0.0	0.0

16	1	6.253E-03	2.541E-02	0.0	0.0	0.0	0.0
15	1	8.401E-03	2.804E-02	0.0	0.0	0.0	0.0
14	1	1.076E-02	2.866E-02	0.0	0.0	0.0	0.0
13	1	8.198E-03	4.357E-03	0.0	0.0	0.0	0.0
12	1	8.746E-03	1.917E-03	0.0	0.0	0.0	0.0
11	1	8.620E-03	4.153E-03	0.0	0.0	0.0	0.0
10	1	8.601E-03	6.984E-03	0.0	0.0	0.0	0.0
9	1	9.290E-03	7.420E-03	0.0	0.0	0.0	0.0
8	1	1.246E-02	8.229E-03	0.0	0.0	0.0	0.0
7	1	1.352E-02	7.559E-03	0.0	0.0	0.0	0.0
6	1	1.120E-02	0.0	0.0	0.0	0.0	0.0
5	1	1.020E-02	0.0	0.0	0.0	0.0	0.0
4	1	9.182E-03	0.0	0.0	0.0	0.0	0.0
3	1	1.047E-02	0.0	0.0	0.0	0.0	0.0
2	1	1.241E-02	0.0	0.0	0.0	0.0	0.0
1	1	1.382E-02	0.0	0.0	0.0	0.0	0.0

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.1000E-01	0.1000E-01	0.1000E-01	0.1000E-01	0.5216E-01	0.6354E 00	0.1000E-01	0.1000E-01	0.1039E-01	0.8679E-01
10	0.1425E 00	0.4187E 00	0.1000E-01	0.2338E-01	0.6573E-01	0.1121E 00	0.1782E 00	0.1663E 00	0.2446E-01	0.3276E-01
20	0.5905E-01	0.9786E-01	0.2869E-01	0.7350E-01	0.1302E-01	0.1000E-01	0.1691E-01	0.3153E-01	0.1000E-01	0.1332E 00
30	0.2400E 00	0.1218E-01	0.1059E-01	0.1000E-01	0.1000E-01	0.1000E-01	0.1000E-01	0.2795E 00	0.1590E 00	0.1047E 00
40	0.2891E-01	0.1000E-01	0.1000E-01							

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT	X-SECT AREA	LOAD COND	AXIAL FORCE
1	0.2795E 00	1	0.1515E 04
2	0.1590E 00	1	0.8292E 03
3	0.1047E 00	1	0.5229E 03
4	0.2891E-01	1	0.1199E 03
5	0.1000E-01	1	0.1138E 02
6	0.1000E-01	1	-0.1322E 02

ANALYSIS OF MEMBRANE ELEMENTS, CONSTRN CODE=

ELEMENT	SHEET THICKNESS	LOAD COND	LOCATION	/---MEMBRANE FORCES IN LOCAL COORDS---/ NXX	NYY	NXY	/--MEMBRANE FORCES IN MATERIAL COORDS-/ N11	N22	N12
1	0.1000E-01	1	CEN	-0.1081E 00	0.9865E 02	0.2051E 01	-0.1081E 00	0.9865E 02	0.2051E 01
2	0.1000E-01	1	CEN	0.4328E 01	0.9909E 02	-0.3600E 01	0.4328E 01	0.9909E 02	-0.3600E 01
3	0.1000E-01	1	CEN	0.1399E 02	0.9641E 02	-0.3956E 01	0.1399E 02	0.9641E 02	-0.3956E 01
4	0.1000E-01	1	CEN	0.2740E 02	0.8576E 02	-0.1237E 02	0.2740E 02	0.8576E 02	-0.1237E 02
5	0.5216E-01	1	CEN	0.6186E 02	0.3209E 03	-0.8390E 02	0.6186E 02	0.3209E 03	-0.8390E 02
6	0.6354E 00	1	CEN	0.2710E 04	0.1333E 04	-0.1574E 04	0.2710E 04	0.1333E 04	-0.1574E 04
7	0.1000E-01	1	CEN	-0.2531E 01	0.1094E 03	-0.2911E 01	-0.2531E 01	0.1094E 03	-0.2911E 01
8	0.1000E-01	1	CEN	0.5541E 01	0.1102E 03	-0.1111E 02	0.5541E 01	0.1102E 03	-0.1111E 02
9	0.1039E-01	1	CEN	0.2107E 02	0.8936E 02	-0.2723E 02	0.2107E 02	0.8936E 02	-0.2723E 02
10	0.8679E-01	1	CEN	0.9475E 02	0.4922E 03	-0.1979E 03	0.9475E 02	0.4922E 03	-0.1979E 03
11	0.1425E 00	1	CEN	0.5742E 02	0.7737E 03	-0.4189E 01	0.5742E 02	0.7737E 03	-0.4189E 01
12	0.4187E 00	1	CEN	0.2184E 04	0.3751E 03	0.6696E 03	0.2184E 04	0.3751E 03	0.6696E 03
13	0.1000E-01	1	CEN	0.8854E 01	0.1089E 03	-0.1601E 02	0.8854E 01	0.1089E 03	-0.1601E 02
14	0.2338E-01	1	CEN	0.1195E 02	0.1741E 03	-0.5163E 02	0.1195E 02	0.1741E 03	-0.5163E 02
15	0.6573E-01	1	CEN	0.3846E 02	0.3460E 03	-0.1532E 03	0.3846E 02	0.3460E 03	-0.1532E 03
16	0.1121E 00	1	CEN	0.3760E 03	0.3410E 03	-0.3335E 03	0.3760E 03	0.3410E 03	-0.3335E 03
17	0.1782E 00	1	CEN	0.8799E 02	0.8042E 03	0.3122E 03	0.8799E 02	0.8042E 03	0.3122E 03
18	0.1663E 00	1	CEN	0.7200E 03	0.1205E 03	0.3147E 03	0.7200E 03	0.1205E 03	0.3147E 03
19	0.2446E-01	1	CEN	0.3091E 02	0.1823E 03	-0.9206E 02	0.3091E 02	0.1823E 03	-0.9206E 02
20	0.3276E-01	1	CEN	0.4868E 02	0.2036E 03	-0.1233E 03	0.4868E 02	0.2036E 03	-0.1233E 03
21	0.5905E-01	1	CEN	0.4930E 02	0.3183E 03	-0.1812E 03	0.4930E 02	0.3183E 03	-0.1812E 03
22	0.9786E-01	1	CEN	-0.1005E 03	0.4594E 03	-0.1462E 03	-0.1005E 03	0.4594E 03	-0.1462E 03
23	0.2869E-01	1	CEN	-0.4218E 01	0.7226E 02	0.3576E 02	-0.4218E 01	0.7226E 02	0.3576E 02
24	0.7350E-01	1	CEN	0.1031E 03	0.7802E 02	0.1297E 03	0.1031E 03	0.7802E 02	0.1297E 03
25	0.1302E-01	1	CEN	0.1468E 00	0.2506E 03	0.2924E 00	0.1468E 00	0.2506E 03	0.2924E 00
26	0.1000E-01	1	CEN	0.4970E 02	0.1706E 03	-0.9502E 02	0.4970E 02	0.1706E 03	-0.9502E 02
27	0.1691E-01	1	CEN	0.7674E 02	0.2174E 03	-0.1512E 03	0.7674E 02	0.2174E 03	-0.1512E 03
28	0.3153E-01	1	CEN	0.6998E 02	0.3019E 03	-0.1605E 03	0.6998E 02	0.3019E 03	-0.1605E 03
29	0.1000E-01	1	CEN	0.2483E 02	0.1044E 03	-0.4359E 02	0.2483E 02	0.1044E 03	-0.4359E 02
30	0.1332E 00	1	CEN	-0.1498E 03	0.8623E 02	0.1194E 03	-0.1498E 03	0.8623E 02	0.1194E 03
31	0.2400E 00	1	CEN	-0.2641E 03	0.2132E 03	0.2418E 03	-0.2641E 03	0.2132E 03	0.2418E 03
32	0.1218E-01	1	CEN	-0.1321E 02	0.2951E 03	-0.3402E 01	-0.1321E 02	0.2951E 03	-0.3402E 01
33	0.1059E-01	1	CEN	0.1244E 03	0.1440E 03	-0.1329E 03	0.1244E 03	0.1440E 03	-0.1329E 03
34	0.1000E-01	1	CEN	0.7477E 02	0.1369E 03	-0.9246E 02	0.7477E 02	0.1369E 03	-0.9246E 02
35	0.1000E-01	1	CEN	0.5766E 02	0.1019E 03	-0.7796E 02	0.5766E 02	0.1019E 03	-0.7796E 02
36	0.1000E-01	1	CEN	0.1810E 02	0.2264E 03	0.1607E 02	0.1810E 02	0.2264E 03	0.1607E 02
37	0.1000E-01	1	CEN	0.1231E 03	0.1181E 03	-0.1082E 03	0.1231E 03	0.1181E 03	-0.1082E 03

****
EVALUATION OF DESIGN NUMBER 7
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.1009E 01	1	33
MIN	0.8785E-01	1	30

MAX DISP RATIOS	LOAD COND	EQN NUMBER
0.1000E 01	1	57
0.3285E 00	1	6

DESIGN IS CRITICAL

STRUCTURAL WEIGHT= 0.2759E 00

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR DISPT. CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	PASS	0.46669E 01
2	PASS	0.27509E 01
3	PASS	0.20246E 01
4	PASS	0.10124E 01
5	ACT	-0.61372E 00
6	ACT	-0.11184E 01
7	PASS	0.31821E 01
8	PASS	0.21367E 01
9	PASS	-0.73782E 00
10	ACT	-0.99731E 00
11	ACT	-0.91146E 00
12	ACT	-0.10865E 01
13	PASS	-0.62892E 00
14	ACT	-0.91557E 00
15	ACT	-0.10384E 01
16	ACT	-0.10167E 01
17	ACT	-0.99288E 00
18	ACT	-0.10153E 01
19	ACT	-0.97199E 00
20	ACT	-0.10245E 01
21	ACT	-0.99244E 00
22	ACT	-0.99983E 00
23	ACT	-0.92391E 00
24	ACT	-0.98679E 00
25	ACT	-0.96531E 00
26	PASS	-0.96643E 00
27	ACT	-0.96663E 00
28	ACT	-0.10263E 01
29	PASS	0.24172E 01
30	ACT	-0.10184E 01
31	ACT	-0.99558E 00
32	PASS	0.91837E 00
33	PASS	0.58773E 00
34	PASS	0.17759E 01
35	PASS	0.12665E 02
36	PASS	0.25223E 01
37	PASS	0.38770E 01
38	ACT	-0.10281E 01
39	ACT	-0.93958E 00
40	ACT	-0.92939E 00
41	ACT	-0.91456E 00
42	PASS	-0.30707E-01
43	PASS	-0.27262E 00

NO. OF ACTIVE DISPLACEMENT CONSTRAINTS ARE 1

O. SHEAR PANEL ELEMENTS

O.1 Shear Lag Problem

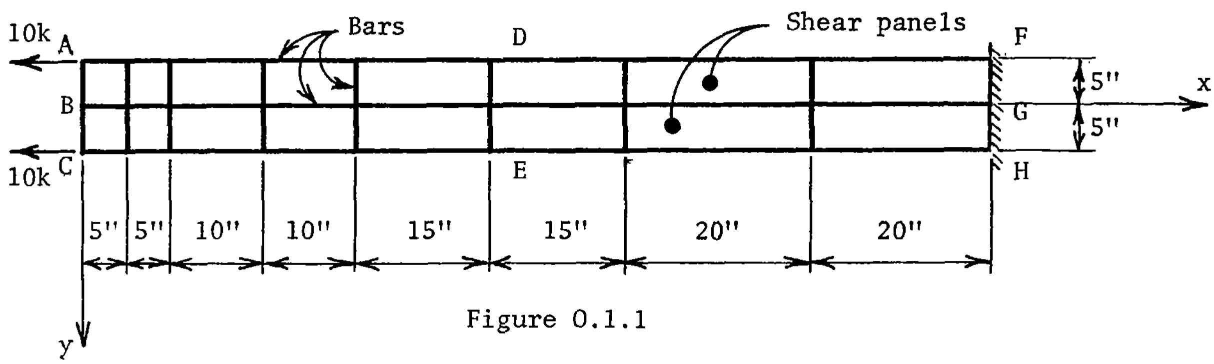

Figure O.1.1

Shear Lag Problem

Figure O.1.1 shows a composite tension member consisting of bars and shear panels. The transverse bars, such as DE, carry no stress; their sole function is to increase the buckling strength of the skin. Consequently, only the shear panels and the longitudinal bars have to be included in the finite element model of the structure.

The material properties of the structure are:

$E = 10 \times 10^6$ psi (Young's modulus),

$\nu = 0.3$ (Poisson's ratio),

$\sigma_t^* = \sigma_c^* = 20{,}000$ psi (allowable normal stress),

$\sigma_s^* = 8{,}000$ psi (allowable shear stress),

$\rho = 0.1$ lb/cu. in. (specific weight).

Apart from stress limits, local buckling of the panels is not allowed, and the following constraints are placed on the displacements at points A, B and C:

$$u_x^* = -0.05'' \quad \text{(in the negative x-direction).}$$

The shear panels are assumed to behave as simply supported plates during buckling.

The design is started with

$t = 0.1$ in. (thickness of sheet),

$A_{AF} = A_{CH} = 1$ sq. in. (cross-sectional area of longitudinals AF and CH),

$A_{BG} = 2$ sq. in. (cross-sectional area of longitudinal BG),

and the following minimum size constraints are imposed:

$t^* = 0.01$ in.,

$A_{AF}^* = A_{CH} = 0.1$ sq. in.,

$A_{BG}^* = 0.2$ sq. in.

Symmetry considerations allow us to model only half of the structure, as shown in Fig. 0.1.2. All elements are sized independently, except for the two shear panels closest to the load, which are required to have the same thickness. The design variable numbers are identified in Fig. 0.1.3.

Two computer runs of the same problem were made. In the first run $\alpha = 0.4$ was used as the relaxation factor. Fourteen redesign

cycles were required for convergence, which is an unusually large number. The final design is given in Fig. O.1.3, and the weight history of the design procedure in Fig. O.1.4. The latter shows a temporary weight increase between design Nos. 1 and 2 which, however, is too small to activate the cut-off criterion $\Delta W/W > \varepsilon$. As in the problem of Sec. M.1.1, the increase is due to a change in the active constraints (displacement to stress).

The second run used $\alpha = 0.3$ in an attempt to reduce the number of redesigns by over-relaxation. This resulted in a much larger increase of structural weight when the change in the active constraints occurred, (see Fig. O.1.4), which in turn caused a termination of the design. The complete history of the second run is given in the computer printout sheets.

This example illustrates that the activation of the cut-off criterion based on weight increase does not necessarily mean that an optimal design has been reached. In order to minimize the possibility of a premature program termination, extensive over-relaxation should be avoided in the first few design cycles, i.e., the first run should use a "normal" value of α (in this case $0.4 \leq \alpha \leq 0.6$). The results of the initial run (optimality indicies, design variables and changes in weight) can then be analyzed, and an appropriate change made in the relaxation factor.

It is important to note that the relaxation factor influences only displacement-constrained designs, since it is not used in the stress ratio method. Consequently, a change in α after the second

redesign would have very little effect on the design history of the current problem.

It should also be mentioned that the minimum size constraints play a major role in this example, despite the fact that only one design variable (design variable No. 9 in Fig. 0.1.3) reaches the minimum value. It is easy to verify that if no size constraints exist, the optimal design is obtained by the removal of all members with the exception of the longitudinal members AF and CH. The half-weight of the resulting structure (determined by the displacement constraint at A) would be 20.0 lb., as compared to 21.38 lb. for the design in Fig. 0.1.3.

Special notes on input-output:

1) Because buckling of the longitudinals was not considered, the moments of inertia were left blank on the bar element data cards (computer replaced the blanks by 10^6 in.^{4}).
2) Although Construction Code No. 1 was specified for the bar elements, Code No. 2 would have served equally well, since the only difference between the two codes lies in the evaluation of the Euler buckling strength.
3) The dimensions of the shear panels (used in local buckling analysis) were left blank on the data cards. Consequently, dimensions in Processed Element Data were calculated by the computer.
4) KSCALE = 1 on Design Control Card indicates that uniform scaling is exact for this problem, the size-stiffness relationship being

$$[K_i] = [k_i]A_i$$

for each element of the structure.

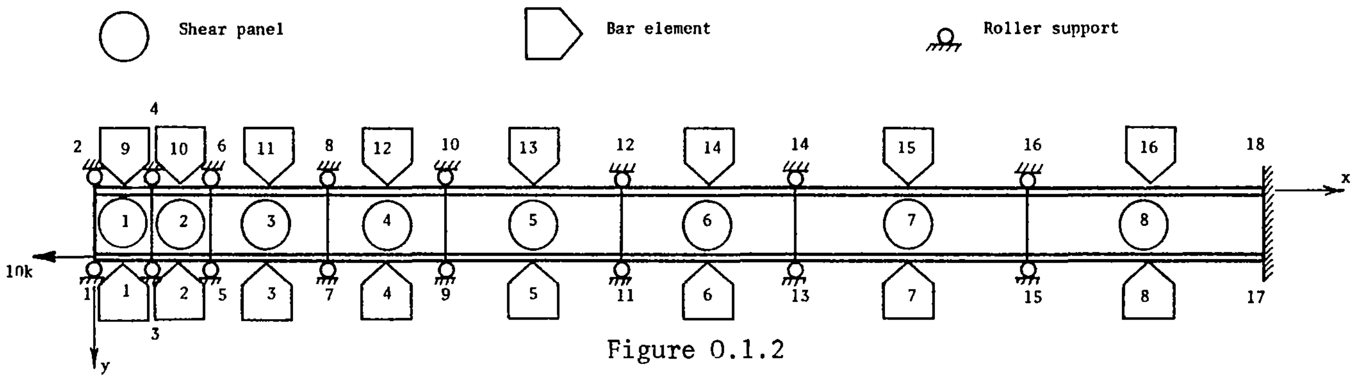

Figure 0.1.2

Finite Element Model for Lower Half of the Structure Showing Element and Node Numbers

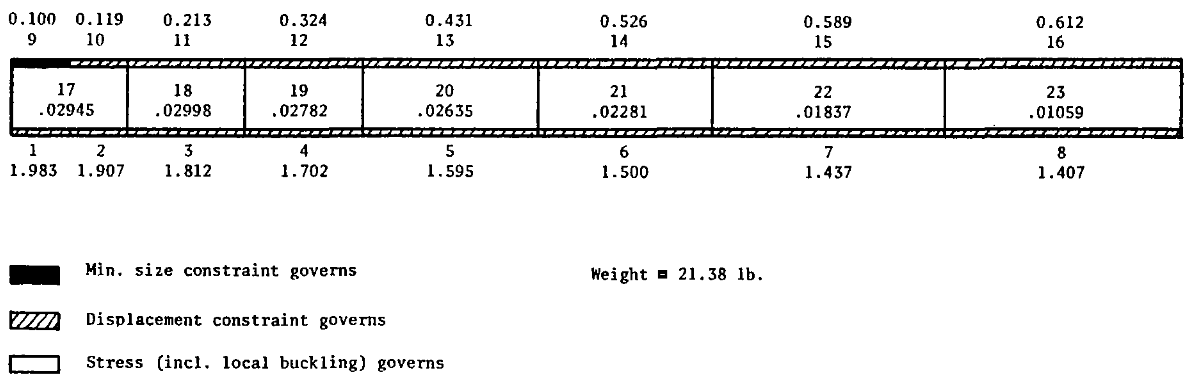

Figure 0.1.3

Optimal Design from the First Run Showing Design Variable Numbers and Final Sizes of Elements

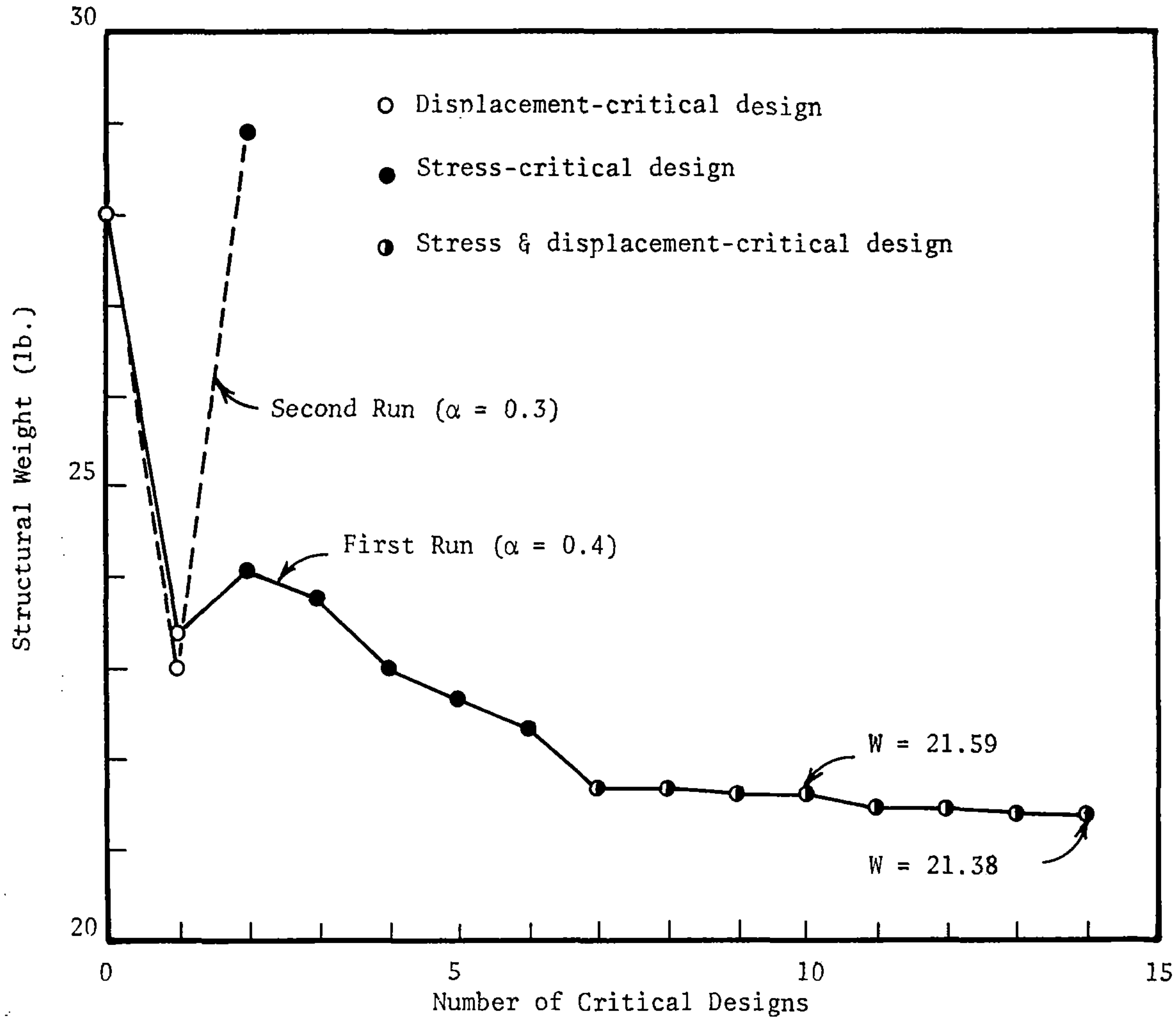

Figure 0.1.4

Weight History of Shear Lag Problem.

```
22950 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
23000  SHEARLAG PROBLEM - STRESS, LOCAL BUCKLING AND DISPLACEMENT CONSTRAINTS
23050    18    2    1   23
23100    15         1 0.025       0.1        1    1    1  0.8       0.3
23150    16  1.0       0.1
23200    23 0.1        0.01
23250     1        -1   -1   -1   -1   -1           -5.
23300     5                               10.       -5.                    2
23350     9                                30.      -5.                    2
23400    13                               60.       -5.                    2
23450    17    1                          100.      -5.                    2
23500     2
23550     6                               10.                              2
23600    10                                30.                             2
23650    14                               60.                              2
23700    18    1                          100.                             2
23750     1   16    1    1    1    1
23800     1    1  0.1
23850           10000000.            20000.    20000.
23900     1      1.
23950
24000
24050
24100
24150     1    1    3    1    1    1
24200     2    3    5    1    1    2
24250     3    5    7    1    1    3
24300     4    7    9    1    1    4
24350     5    9   11    1    1    5
24400     6   11   13    1    1    6
24450     7   13   15    1    1    7
24500     8   15   17    1    1    8
24550     9    2    4    1    1    9
24600    10    4    6    1    1   10
24650    11    6    8    1    1   11
24700    12    8   10    1    1   12
24750    13   10   12    1    1   13
24800    14   12   14    1    1   14
24850    15   14   16    1    1   15
24900    16   16   18    1    1   16
24950     4    8    1    1    1
25000     1    1   0.1
25050            10000000.  0.3      8000.
25100
25150
25200
25250     2    3    5    6    4    1   17    1                                   2
25300     3    5    7    8    6    1   18    1
25350     4    7    9   10    8    1   19    1
25400     5    9   11   12   10    1   20    1
25450     6   11   13   14   12    1   21    1
25500     7   13   15   16   14    1   22    1
25550     8   15   17   18   16    1   23    1
25600
25650    1   1                                -0.05
25700    2   1                                -0.05
25750
25800     1    1 -10000.
25850
25900 123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
25950
```

Echo of Input Cards for Second Run

SHEARLAG PROBLEM - STRESS, LOCAL BUCKLING AND DISPLACEMENT CONSTRAINTS

NUMBER OF NODAL POINTS = 18
NUMBER OF ELEMENT TYPES = 2
NUMBER OF LOAD CASES = 1
NUMBER OF DES. VARIABLES = 23

DESIGN CONTROL DATA

NCYCL = 15
KSCALE= 1
DELTA = 0.2500E-01
EPSIL = 0.1000E 00
KDISP = 1
OMEGA = 0.80000
ALPA = 0.30000

DESIGN VARIABLE INPUT DATA

DESIGN VARIABLE NUMBER	INITIAL VALUE	MIN ALLOWABLE VALUE
1	0.1000E 01	0.1000E 00
2	0.1000E 01	0.1000E 00
3	0.1000E 01	0.1000E 00
4	0.1000E 01	0.1000E 00
5	0.1000E 01	0.1000E 00
6	0.1000E 01	0.1000E 00
7	0.1000E 01	0.1000E 00
8	0.1000E 01	0.1000E 00
9	0.1000E 01	0.1000E 00
10	0.1000E 01	0.1000E 00
11	0.1000E 01	0.1000E 00
12	0.1000E 01	0.1000E 00
13	0.1000E 01	0.1000E 00
14	0.1000E 01	C.1000E 00
15	0.1000E 01	0.1000E 00
16	0.1000E 01	0.1000E 00
17	0.1000E 00	0.1000E-01
18	0.1000E 00	0.1000E-01
19	0.1000E 00	0.1000E-01
20	0.1000E 00	0.1000E-01
21	0.1000E 00	0.1000E-01
22	0.1000E 00	0.1000E-01
23	0.1000E 00	0.1000E-01

NODAL POINT INPUT DATA

NODE	BOUNDARY CONDITION CODES						/------NODAL POINT COORDINATES--------/				
NUMBER	X	Y	Z	XX	YY	ZZ	X	Y	Z		T
1	0	-1	-1	-1	-1	-1	0.000	-5.000	0.000	0	0.000
5	0	0	0	0	0	0	10.000	-5.000	0.000	2	0.000
9	0	0	0	0	0	0	30.000	-5.000	0.000	2	0.000
13	0	0	0	0	0	0	60.000	-5.000	0.000	2	0.000
17	1	0	0	0	0	0	100.000	-5.000	0.000	2	0.000

Computer Printout for Second Run

2	0	0	0	0	0	0	0.000	0.000	0.000	0	0.000
6	0	0	0	0	0	0	10.000	0.000	0.000	2	0.000
10	0	0	0	0	0	0	30.000	0.000	0.000	2	0.000
14	0	0	0	0	0	0	60.000	0.000	0.000	2	0.000
18	1	0	0	0	0	0	100.000	0.000	0.000	2	0.000

GENERATED NODAL DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	/------NODAL PCINT COORDINATES--------/ X	Y	Z	T
1	0	-1	-1	-1	-1	-1	0.000	-5.000	0.000	0.000
2	0	-1	-1	-1	-1	-1	0.000	0.000	0.000	0.000
3	0	-1	-1	-1	-1	-1	5.000	-5.000	0.000	0.000
4	0	-1	-1	-1	-1	-1	5.000	0.000	0.000	0.000
5	0	-1	-1	-1	-1	-1	10.000	-5.000	0.000	0.000
6	0	-1	-1	-1	-1	-1	10.000	0.000	0.000	0.000
7	0	-1	-1	-1	-1	-1	20.000	-5.000	0.000	0.000
8	0	-1	-1	-1	-1	-1	20.000	0.000	0.000	0.000
9	0	-1	-1	-1	-1	-1	30.000	-5.000	0.000	0.000
10	0	-1	-1	-1	-1	-1	30.000	0.000	0.000	0.000
11	0	-1	-1	-1	-1	-1	45.000	-5.000	0.000	0.000
12	0	-1	-1	-1	-1	-1	45.000	0.000	0.000	0.000
13	0	-1	-1	-1	-1	-1	60.000	-5.000	0.000	0.000
14	0	-1	-1	-1	-1	-1	60.000	0.000	0.000	0.000
15	0	-1	-1	-1	-1	-1	80.000	-5.000	0.000	0.000
16	0	-1	-1	-1	-1	-1	80.000	0.000	0.000	0.000
17	1	-1	-1	-1	-1	-1	100.000	-5.000	0.000	0.000
18	1	-1	-1	-1	-1	-1	100.000	0.000	0.000	0.000

EQUATION NUMBERS

N	X	Y	Z	XX	YY	ZZ
1	1	0	0	0	0	0
2	2	0	0	0	0	0
3	3	0	0	0	0	0
4	4	0	0	0	0	0
5	5	0	0	0	0	0
6	6	0	0	0	0	0
7	7	0	0	0	0	0
8	8	0	0	0	0	0
9	9	0	0	0	0	0
10	10	0	0	0	0	0
11	11	0	0	0	0	0
12	12	0	0	0	0	0
13	13	0	0	0	0	0
14	14	0	0	0	0	0
15	15	0	0	0	0	0
16	16	0	0	0	0	0
17	0	0	0	0	0	0
18	0	0	0	0	0	0

NUMBER OF TRUSS ELEMENTS = 16
CONSTRUCTION CODE = 1
NUMBER OF MATERIALS = 1
NUMBER OF TEMPS FOR WHICH MATL PROPS GIVEN= 1
NUMBER OF DIFFERENT GEOMETRIES PROPS GIVEN= 1

MATERIAL PROPERTY CARDS

MATERIAL NUMBER	NUMBER OF TEMPS	SPECIFIC WEIGHT	TEMP	YOUNGS MODULUS	COEFFT OF THERM EXPAN	/--ALLOWABLE STRESSES--/ TENSION	COMPRESSION
1	1	0.1000E 00	0.0000E 00	0.1000E 08	0.0000E 00	0.2000E 05	0.2000E 05

GEOMETRIC PROPERTY CARDS

GEOMETRY NUMBER	X-SECT AREA	/--MOMENTS OF INERTIA--/ YY	ZZ
1	0.1000D 01	0.1000E 07	0.1000E 07

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
Y-DIR	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
Z-DIR	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
TEMP	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00

PROCESSED ELEMENT DATA

ELEMENT NUMBER	/-NODE NOS-/ I	J	/--ELEMENT ID NOS-/ MATL	GEOMY	D VAR	DESIGN VAR FRACTION	REFERENCE TEMP	END FIXITY COEFFICIENTS YY	ZZ	BAND WIDTH
1	1	3	1	1	1	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
2	3	5	1	1	2	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
3	5	7	1	1	3	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
4	7	9	1	1	4	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
5	9	11	1	1	5	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
6	11	13	1	1	6	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
7	13	15	1	1	7	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
8	15	17	1	1	8	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	1
9	2	4	1	1	9	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
10	4	6	1	1	10	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
11	6	8	1	1	11	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
12	8	10	1	1	12	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
13	10	12	1	1	13	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
14	12	14	1	1	14	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
15	14	16	1	1	15	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	3
16	16	18	1	1	16	0.1000E 01	0.0000D 00	0.1000D 01	0.1000D 01	1

NUMBER OF SHEAR PANEL ELEMENTS = 8
CONSTRUCTION CODE = 1
NUMBER OF MATERIALS = 1
NUMBER OF TEMPS FOR WHICH MATL PROPS GIVEN= 1

MATERIAL PROPERTY CARDS

MATERIAL NUMBER	NUMBER OF TEMPS	SPECIFIC WEIGHT	TEMP	YOUNGS MODULUS	POISSN RATIO	ALLOWABLE SHEAR
1	1	0.1000E 00	0.0000E 00	0.1000E 08	0.3000E 00	0.8000E 04

ELEMENT LOAD MULTIPLIERS

	A	B	C	D
X-DIR	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
Y-DIR	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00
Z-DIR	0.000000D 00	0.000000D 00	0.000000D 00	0.000000D 00

PROCESSED ELEMENT DATA

ELEMENT NUMBER	/-------NODE NOS- I	J	K	-----// L	--EL ID NOS-/ MATL	D VAR	BOUND CODE	DES VAR FRACTION	/--EFFECT PANEL DIMNS--/ LONGER	SHORTER	BAND WIDTH
1	1	3	4	2	1	17	1	0.1000E 01	0.5000D 01	0.5000D 01	4
2	3	5	6	4	1	17	1	0.1000E 01	0.5000D 01	0.5000D 01	4
3	5	7	8	6	1	18	1	0.1000E 01	0.1000D 02	0.5000D 01	4
4	7	9	10	8	1	19	1	0.1000E 01	0.1000D 02	0.5000D 01	4
5	9	11	12	10	1	20	1	0.1000E 01	0.1500D 02	0.5000D 01	4
6	11	13	14	12	1	21	1	0.1000E 01	0.1500D 02	0.5000D 01	4
7	13	15	16	14	1	22	1	0.1000E 01	0.2000D 02	0.5000D 01	4
8	15	17	18	16	1	23	1	0.1000E 01	0.2000D 02	0.5000D 01	2

STRUCTURE LOAD CASE	STRUCTURE LOAD MULTIPLIERS A	B	C	D
1	0.000	0.000	0.000	0.000

NODAL DISPLACEMENT/ROTATION CONSTRAINTS

NODE NO.	LOAD/ CASE	DX	DY	DZ	RX	RY	RZ	-DX	-DY	-DZ	-RX	-RY	-RZ
		-----MAX.ALLOWABLE DISPLACEMENTS AND ROTATIONS-----/											
1	1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.05000	0.00000	0.00000	0.00000	0.00000	0.00000
2	1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	-0.05000	0.00000	0.00000	0.00000	0.00000	0.00000

NODAL POINT LOADS

NODE NO.	LOAD CASE	APPLIED LOADS RX	RY	RZ	MX	MY	MZ
1	1	-0.100E 05	0.000E 00	0.000E 00	0.000E 00	0.000E 00	0.000E 00

TOTAL NUMBER OF EQUATIONS = 16

```
BANDWIDTH                    =    4
NUMBER OF EQUATIONS IN A BLOCK =   16
NUMBER OF BLOCKS             =    1
```

****
ANALYSIS OF DESIGN NUMBER 0
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
18	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
17	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
16	1	-1.000E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
15	1	-1.000E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
14	1	-2.000E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
13	1	-2.000E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
12	1	-2.750E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
11	1	-2.750E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
10	1	-3.494E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
9	1	-3.506E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
8	1	-3.974E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
7	1	-4.026E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
6	1	-4.388E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
5	1	-4.612E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
4	1	-4.538E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
3	1	-4.962E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	1	-4.597E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
1	1	-5.403E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01
10	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 01	0.1000E 00	0.1000E 00	0.1000E 00	0.1000E 00
20	0.1000E 00	0.1000E 00	0.1000E 00							

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT X-SECT AREA LOAD COND AXIAL FORCE

1	0.1000E 01	1	0.8817E 04
2	0.1000E 01	1	0.7010E 04
3	0.1000E 01	1	0.5856E 04
4	0.1000E 01	1	0.5201E 04
5	0.1000E 01	1	0.5040E 04
6	0.1000E 01	1	0.5002E 04
7	0.1000E 01	1	0.5000E 04
8	0.1000E 01	1	0.5000E 04
9	0.1000E 01	1	0.1183E 04
10	0.1000E 01	1	0.2990E 04
11	0.1000E 01	1	0.4144E 04
12	0.1000E 01	1	0.4799E 04
13	0.1000E 01	1	0.4961E 04
14	0.1000E 01	1	0.4999E 04
15	0.1000E 01	1	0.5000E 04
16	0.1000E 01	1	0.5000E 04

ANALYSIS OF SHEAR PANELS, CONSTRN CODE= 1

		LOAD	/--------------SHEAR FLOW AT NODES-------------/				AVERAGE
ELEMENT	THICKNESS	COND	I	J	K	L	SHEAR FLOW
1	0.1000E 00	1	0.4734E 03	0.4734E 03	0.4734E 03	0.4734E 03	0.4734E 03
2	0.1000E 00	1	0.2493E 03	0.2493E 03	0.2493E 03	0.2493E 03	0.2493E 03
3	0.1000E 00	1	0.1062E 03	0.1062E 03	0.1062E 03	0.1062E 03	0.1062E 03
4	0.1000E 00	1	0.2489E 02	0.2489E 02	0.2489E 02	0.2489E 02	0.2489E 02
5	0.1000E 00	1	0.4897E 01	0.4897E 01	0.4897E 01	0.4897E 01	0.4897E 01
6	0.1000E 00	1	0.1767E 00	0.1767E 00	0.1767E 00	0.1767E 00	0.1767E 00
7	0.1000E 00	1	0.5175E-02	0.5175E-02	0.5175E-02	0.5175E-02	0.5175E-02
8	0.1000E 00	1	-0.7823E-03	-0.7823E-03	-0.7823E-03	-0.7823E-03	-0.7823E-03

****
EVALUATION OF DESIGN NUMBER 0
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.5917E 00	1	17
MIN	0.1000E 00	0	21

MAX DISP RATIOS	LOAD COND	EQN NUMBER
-0.9194E 00	1	2
-0.1081E 01	1	1

UNIFORM SCALING OPERATION FOLLOWS

SCALE FACTOR IS 1.081AND DETERMINED BY DISPLACEMENT CONSTRAINTS

DESIGN VARIABLES OF SCALED (CRITICAL) DESIGN ARE

VALUES OF DESIGN VARIABLES

1	2	3	4	5	6	7	8	9	10

0	0.1081E 01	0.1081E 01	0.1081E 01	0.1081E 01	0.1081E 01	0.1081E 01	0.1081E 01	0.1081E 01	0.1081E 01	0.1081E 01
10	0.1081E 01	0.1081E 01	0.1081E 01	0.1081E 01	0.1081E 01	0.1081E 01	0.1081E 00	0.1081E 00	0.1081E 00	0.1081E 00
20	0.1081E 00	0.1081E 00	0.1081E 00							

STRUCTURAL WEIGHT= 0.2702E 02

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR DISPT. CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	ACT	-0.25531E 01
2	ACT	-0.16140E 01
3	ACT	-0.11263E 01
4	ACT	-0.88839E 00
5	ACT	-0.83420E 00
6	ACT	-0.82166E 00
7	ACT	-0.82121E 00
8	ACT	-0.82120E 00
9	ACT	-0.46002E-01
10	ACT	-0.29367E 00
11	ACT	-0.56410E 00
12	ACT	-0.75659E 00
13	ACT	-0.80826E 00
14	ACT	-0.82072E 00
15	ACT	-0.82118E 00
16	ACT	-0.82120E 00
17	ACT	-0.12222E 01
18	ACT	-0.96234E-01
19	ACT	-0.52886E-02
20	ACT	-0.20469E-03
21	ACT	-0.26384E-06
22	ACT	-0.19791E-09
23	ACT	-0.56011E-11

NO. OF ACTIVE DISPLACEMENT CONSTRAINTS ARE 1

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******************************
ANALYSIS OF DESIGN NUMBER    1
******************************
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NODAL DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
18	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
17	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
16	1	-1.057E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
15	1	-1.059E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
14	1	-2.109E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
13	1	-2.123E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
12	1	-2.885E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
11	1	-2.932E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
10	1	-3.634E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
9	1	-3.770E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
8	1	-4.109E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
7	1	-4.346E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
6	1	-4.590E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
5	1	-4.889E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
4	1	-4.815E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
3	1	-5.133E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	1	-4.935E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
1	1	-5.336E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.2256E 01	0.1545E 01	0.1176E 01	0.9962E 00	0.9552E 00	0.9457E 00	0.9454E 00	0.9454E 00	0.3590E 00	0.5463E 00
10	0.7509E 00	0.8965E 00	0.9356E 00	0.9450E 00	0.9454E 00	0.9454E 00	0.1249E 00	0.3970E-01	0.3282E-01	0.3243E-01
20	0.3242E-01	0.3242E-01	0.3242E-01							

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT   X-SECT AREA   LOAD COND   AXIAL FORCE

1	0.2256E 01	1	0.9138E 04
2	0.1545E 01	1	0.7536E 04
3	0.1176E 01	1	0.6386E 04
4	0.9962E 00	1	0.5741E 04
5	0.9552E 00	1	0.5334E 04
6	0.9457E 00	1	0.5105E 04
7	0.9454E 00	1	0.5028E 04
8	0.9454E 00	1	0.5005E 04
9	0.3590E 00	1	0.8616E 03
10	0.5463E 00	1	0.2464E 04
11	0.7509E 00	1	0.3614E 04
12	0.8965E 00	1	0.4259E 04
13	0.9356E 00	1	0.4667E 04
14	0.9450E 00	1	0.4895E 04
15	0.9454E 00	1	0.4972E 04
16	0.9454E 00	1	0.4995E 04

ANALYSIS OF SHEAR PANELS, CONSTRN CODE= 1

ELEMENT	THICKNESS	LOAD COND	/---SHEAR FLOW AT NODES--- I	J	K	L/	AVERAGE SHEAR FLOW
1	0.1249E 00	1	0.3447E 03	0.3447E 03	0.3447E 03	0.3447E 03	0.3447E 03
2	0.1249E 00	1	0.2962E 03	0.2962E 03	0.2962E 03	0.2962E 03	0.2962E 03
3	0.3970E-01	1	0.8192E 02	0.8192E 02	0.8192E 02	0.8192E 02	0.8192E 02
4	0.3282E-01	1	0.4717E 02	0.4717E 02	0.4717E 02	0.4717E 02	0.4717E 02
5	0.3243E-01	1	0.2284E 02	0.2284E 02	0.2284E 02	0.2284E 02	0.2284E 02
6	0.3242E-01	1	0.7594E 01	0.7594E 01	0.7594E 01	0.7594E 01	0.7594E 01
7	0.3242E-01	1	0.2023E 01	0.2023E 01	0.2023E 01	0.2023E 01	0.2023E 01
8	0.3242E-01	1	0.2719E 00	0.2719E 00	0.2719E 00	0.2719E 00	0.2719E 00

****
EVALUATION OF DESIGN NUMBER 1
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.8347E 00	1	19
MIN	0.2026E 00	1	1

MAX DISP RATIOS	LOAD COND	EQN NUMBER
-0.9871E 00	1	2
-0.1067E 01	1	1

UNIFORM SCALING OPERATION FOLLOWS

SCALE FACTOR IS 1.067AND DETERMINED BY DISPLACEMENT CONSTRAINTS

DESIGN VARIABLES OF SCALED (CRITICAL) DESIGN ARE

VALUES OF DESIGN VARIABLES

1 2 3 4 5 6 7 8 9 10

0	0.2407E 01	0.1649E 01	0.1255E 01	0.1063E 01	0.1019E 01	0.1009E 01	0.1009E 01	0.1009E 01	0.3831E 00	0.5830E 00
10	0.8013E 00	0.9567E 00	0.9984E 00	0.1008E 01	0.1009E 01	0.1009E 01	0.1333E 00	0.4236E-01	0.3502E-01	0.3461E-01
20	0.3460E-01	0.3460E-01	0.3460E-01							

STRUCTURAL WEIGHT= 0.2297E 02

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR DISP1. CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	ACT	-0.60437E 00
2	ACT	-0.87594E 00
3	ACT	-0.10854E 01
4	ACT	-0.12226E 01
5	ACT	-0.11479E 01
6	ACT	-0.10730E 01
7	ACT	-0.10415E 01
8	ACT	-0.10320E 01
9	ACT	-0.21210E 00
10	ACT	-0.74871E 00
11	ACT	-0.85274E 00
12	ACT	-0.83105E 00
13	ACT	-0.91594E 00
14	ACT	-0.98775E 00
15	ACT	-0.10184E 01
16	ACT	-0.10278E 01
17	ACT	-0.63386E 00
18	PASS	-0.40766E 00
19	PASS	-0.19774E 00
20	PASS	-0.47487E-01
21	PASS	-0.52532E-02
22	ACT	-0.37277E-03
23	ACT	-0.67353E-05

NO. OF ACTIVE DISPLACEMENT CONSTRAINTS ARE 1

****
ANALYSIS OF DESIGN NUMBER 2
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
18	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
17	1	0.000E-01	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
16	1	-9.511E-03	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
15	1	-9.907E-03	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
14	1	-1.895E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
13	1	-1.988E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
12	1	-2.594E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
11	1	-2.743E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
10	1	-3.284E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
9	1	-3.501E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
8	1	-3.738E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
7	1	-4.003E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
6	1	-4.183E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
5	1	-4.514E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
4	1	-4.408E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
3	1	-4.775E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
2	1	-4.625E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01
1	1	-5.040E-02	0.000E-01	0.000E-01	0.0000E-01	0.0000E-01	0.0000E-01

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.1740E 01	0.1506E 01	0.1330E 01	0.1229E 01	0.1125E 01	0.1061E 01	0.1038E 01	0.1031E 01	0.1718E 00	0.4805E 00
10	0.7187E 00	0.8435E 00	0.9396E 00	0.9998E 00	0.1022E 01	0.1028E 01	0.9910E-01	0.3293E-01	0.2739E-01	0.2218E-01
20	0.1536E-01	0.1039E-01	0.1038E-01							

ANALYSIS OF TRUSS ELEMENTS, CONSTRN CODE= 1

ELEMENT X-SECT AREA LOAD COND AXIAL FORCE

1	0.1740E 01	1	0.9254E C4
2	0.1506E 01	1	0.7843E 04
3	0.1330E 01	1	0.6800E 04
4	0.1229E 01	1	0.6169E 04
5	0.1125E 01	1	0.5681E C4
6	0.1061E 01	1	0.5339E 04
7	0.1038E 01	1	0.5178E C4
8	0.1031E 01	1	0.5109E C4
9	0.1718E 00	1	0.7459E C3
10	0.4805E 00	1	0.2157E C4
11	0.7187E 00	1	0.3200E 04
12	0.8435E 00	1	0.3831E C4
13	0.9396E 00	1	0.4319E C4
14	0.9998E 00	1	0.4661E 04
15	0.1022E 01	1	0.4822E 04
16	0.1028E 01	1	0.4891E 04

ANALYSIS OF SHEAR PANELS, CONSTRN CODE= 1

ELEMENT	THICKNESS	LOAD COND	/--------------SHEAR FLOW AT NODES I	J	K	L-------------/	AVERAGE SHEAR FLOW
1	0.9910E-01	1	0.2984E 03	0.2984E 03	0.2984E 03	0.2984E 03	0.2984E 03
2	0.9910E-01	1	0.2661E 03	0.2661E 03	0.2661E 03	0.2661E C3	0.2661E 03
3	0.3293E-01	1	0.7548E 02	0.7548E 02	0.7548E 02	0.7548E 02	0.7548E 02
4	0.2739E-01	1	0.5C79E 02	0.5079E 02	0.5079E 02	0.5079E 02	0.5079E 02
5	0.2218E-01	1	0.3124E 02	0.3124E 02	0.3124E 02	0.3124E 02	0.3124E 02
6	0.1536E-01	1	0.1433E 02	0.1433E 02	0.1433E 02	0.1433E 02	0.1433E 02
7	0.1039E-01	1	0.5316E 01	0.5316E 01	0.5316E 01	0.5316E 01	0.5316E 01
8	0.1038E-01	1	0.1581E 01	0.1581E 01	0.1581E 01	0.1581E 01	0.1581E 01

****
EVALUATION OF DESIGN NUMBER 2
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.1328E 01	1	22
MIN	0.2226E 00	1	11

MAX DISP RATIOS	LOAD COND	EQN NUMBER
-0.9249E 00	1	2
-0.1008E 01	1	1

UNIFORM SCALING OPERATION FOLLOWS

SCALE FACTOR IS 1.328AND DETERMINED BY STRESS CONSTRAINTS

DESIGN VARIABLES OF SCALED (CRITICAL) DESIGN ARE

VALUES OF DESIGN VARIABLES

1 2 3 4 5 6 7 8 9 10

0.1.20

0	0.2311E 01	0.2000E 01	0.1767E 01	0.1632E 01	0.1494E 01	0.1409E 01	0.1379E 01	0.1370E 01	0.2282E 00	0.6381E 00
10	0.9546E 00	0.1120E 01	0.1248E 01	0.1328E 01	0.1357E 01	0.1366E 01	0.1316E 00	0.4373E-01	0.3638E-01	0.2946E-01
20	0.2040E-01	0.1380E-01	0.1378E-01							

STRUCTURAL WEIGHT= 0.2893E 02

REDESIGN OPERATION FOLLOWS
TERMINAL DESIGN---LIGHTEST CRITICAL DESIGN IS DESIGN NUMBER 1

P. PLATE ELEMENTS

P.1 Square Plate with Clamped Edges - Problem 1

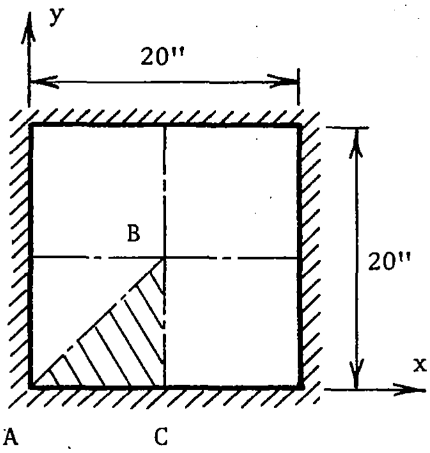

Figure P.1.1

Uniformly Loaded Plate.
(Shaded region is modelled by finite elements)

We consider a square, clamped plate subjected to a uniform normal pressure of 16 psi. Multiple symmetries permit us to treat only one-eighth of the plate as indicated in Fig. P.1.1; the finite element mesh employed in the design is shown in Fig. P.1.2. The clamped boundary conditions on AC and the symmetry requirements on BC are imposed by the appropriate motion code on Nodal Point Data cards, but Boundary Elements must be used to enforce the symmetry condition (vanishing slope) on the skewed line AB.

The data used in the design is

$E = 10.5 \times 10^6$ psi (Young's modulus)

$\nu = 0.3$ (Poisson's ratio)

$\sigma_t^* = \sigma_c^* = 12{,}000$ psi (Allowable stresses)

$\rho = 0.1$ lb/cu. in. (Specific weight)

$u_z^* = 0.1$ in. at center of plate (allowable displacement).

Rotational springs are used for the boundary elements, with a spring constant of 2×10^6 in. lb/rad. each. This value is roughly 100 times

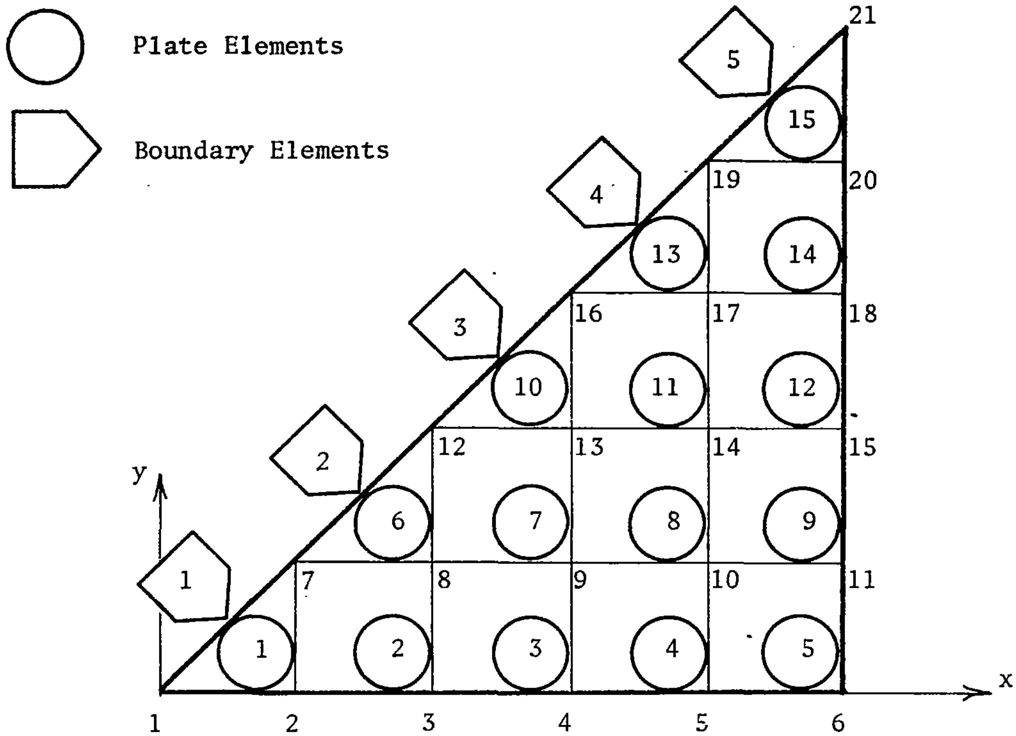

Figure P.1.2

Finite Element Mesh Showing Element and Node Numbers

higher than the corresponding stiffness $k = M/\theta$ (see Fig. P.1.3) of a typical element in the initial design. As most elements become thinner during redesign, the boundary elements become relatively stiffer and more effective in enforcing the symmetry condition on AB.

The initial design is a uniform plate of thickness 0.3 in. No minimum size constraints were employed in the design. All plate elements were sized independently.

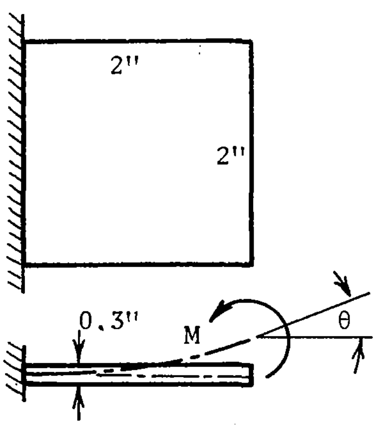

Figure P.1.3
Computation of Rotational Stiffness of a Plate Element.

Table P.1.1 shows that the first four redesigns were completely governed by the displacement constraint. It is very likely this portion of the design could be speeded up by over-relaxation (the "normal" value of the relaxation factor, $\alpha = 0.75$, was used). From the fifth design onwards, an increasing number of stress constraints become active, and slow down the convergence. In fact, an increase in weight occurs in design No. 7, which is not overcome in the next two redesign cycles. The use of over-relaxation would be ineffective in this case due to the presence of stress constraints.

Slow convergence is not uncommon in design problems where a gradual change occurs in the critical constraints. It is seldom practical to run such problems until the optimality criteria are reached. It has been our experience, however, that the structural weight does not decrease much after the first 4-6 redesigns, although the design variables may change considerably. In this particular problem, for example, we would not hesitate to adopt design No. 6 as the final design.

Special notes on input-output:

1) The normal pressure was specified on the element cards, not as nodal point loads (see Thin Plate/Shell Element Data).
2) Since the stiffness matrix of each plate element has the form $[K_i] = [k_i]A^3$ (A is the plate thickness), uniform scaling was declared to be an exact operation by the use of KSCALE = 3 (see Design Control Data). The scaling factor is actually a little in error due to the presence of the boundary elements with finite stiffness constants.

3) The isotropic von Mises yield criterion was employed for stress constraints. By leaving the allowable shear stress σ_s^* blank on the material property cards, $\sigma_s^* = \sigma_t^*/\sqrt{3}$ was used in the design (see Material Property Table).

Element	Critical, Scaled Designs (thickness in inches)									
	0	1	2	3	4	5	6	7	8	9
1	.3225	.2446	.1852	.1399	.1054	.0792	.0595	.0500	.0449	.0393
2	.3225	.2562	.2024	.1579	.1214	.0923	.0696	.0586	.0512	.0438
3	.3225	.2826	.2471	.2102	.1719	.1360	.1052	.1096	.1282	.1468
4	.3225	.3183	.3223	.3182	.2986	.2680	.2305	.2315	.2280	.2140
5	.3225	.3445	.3894	.4350	.4673	.4954	.5212	.6089	.6457	.6381
6	.3225	.2603	.2091	.1658	.1285	.0973	.0723	.0708	.0764	.0779
7	.3225	.2637	.2176	.1820	.1504	.1205	.0939	.0803	.0705	.0605
8	.3225	.2611	.2199	.2010	.1889	.1709	.1462	.1329	.1191	.1032
9	.3225	.2571	.2211	.2232	.2482	.2810	.3162	.3882	.4201	.4197
10	.3225	.2637	.2197	.1899	.1686	.1488	.1287	.1321	.1376	.1347
11	.3225	.2673	.2235	.2027	.1890	.1716	.1497	.1455	.1454	.1425
12	.3225	.2626	.2210	.2057	.2023	.1938	.1778	.1801	.1808	.1742
13	.3225	.2880	.2857	.2960	.2933	.2785	.2566	.2608	.2575	.2433
14	.3225	.3185	.3361	.3575	.3581	.3437	.3212	.3326	.3332	.3191
15	.3225	.4226	.4889	.5130	.5055	.4816	.4499	.4679	.4706	.4519
Wt. (lb.)	.01612	.01427	.01318	.01258	.01199	.01126	.01046	.01104	.01126	.01094

Table P.1.1

Design History of Element Thicknesses and Total Structural Weight

(Underlined thicknesses are governed by stress constraints.)

```
123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H
  CLAMPED SQUARE PLATE - STRESS AND DISP. CONSTRAINTS
   21    2    1   15
   10         3 0.025      0.1         1    1    1  0.8      0.75
   15  0.3
    1   -1   -1   -1   -1    1   -1
    2                               2.0
    6              1    1    1       10.                            1
    7                               2.0       2.0
   11                        1       10.       2.0                  1
   12                                4.0      4.0
   15                        1       10.       4.0                  1
   16                                6.0      6.0
   18                        1      10.        6.0                  1
   19                                8.0       8.0
   20                        1      10.         8.0
   21                        1      10.        10.
    6   15    1    1    1
    1    1 0.1
          10500000.  0.3                 12000.    12000.
  1.




    1    1    2    7         1         1 16.
    2    2    3    8    7    1         2 16.
    3    3    4    9    8    1         3 16.
    4    4    5   10    9    1         4 16.
    5    5    6   11   10    1         5 16.
    6    7    8   12         1         6 16.
    7    8    9   13   12    1         7 16.
    8    9   10   14   13    1         8 16.
    9   10   11   15   14    1         9 16.
   10   12   13   16         1        10 16.
   11   13   14   17   16    1        11 16.
   12   14   15   18   17    1        12 16.
   13   16   17   19         1        13 16.
   14   17   18   20   19    1        14 16.
   15   19   20   21         1        15 16.
    7    5

    1    7    1                        1                                2000000.
    2   12    7                        1                                2000000.
    3   16   12                        1                                2000000.
    4   19   16                        1                                2000000.
    5   21   19                        1                                2000000.
  1.0
  21   1              .1                                  -.1


123456789A123456789B123456789C123456789D123456789E123456789F123456789G123456789H

```

Echo of Input Cards

CLAMPED SQUARE PLATE - STRESS AND DISP. CONSTRAINTS

```
NUMBER OF NODAL POINTS   =   21
NUMBER OF ELEMENT TYPES  =    2
NUMBER OF LOAD CASES     =    1
NUMBER OF DES. VARIABLES =   15
```

DESIGN CONTROL DATA

```
NCYCL =    10
NSCALE=     3
DELTA =  0.2500E-01
EPSIL =  0.1000E 00
KDISP =     1
OMEGA =   0.80000
ALPA  =   0.75000
```

DESIGN VARIABLE INPUT DATA

DESIGN VARIABLE NUMBER	INITIAL VALUE	MIN ALLOWABLE VALUE
1	0.3000E 00	0.0
2	0.3000E 00	0.0
3	0.3000E 00	0.0
4	0.3000E 00	0.0
5	0.3000E 00	0.0
6	0.3000E 00	0.0
7	0.3000E 00	0.0
8	0.3000E 00	0.0
9	0.3000E 00	0.0
10	0.3000E 00	0.0
11	0.3000E 00	0.0
12	0.3000E 00	0.0
13	0.3000E 00	0.0
14	0.3000E 00	0.0
15	0.3000E 00	0.0

NODAL POINT INPUT DATA

NODE NUMBER	BOUNDARY CONDITION CODES X	Y	Z	XX	YY	ZZ	/------NODAL POINT COORDINATES--------/ X	Y	Z		T
1	-1	-1	-1	-1	1	-1	0.0	0.0	0.0	0	0.0
2	0	0	0	0	0	0	2.000	0.0	0.0	0	0.0
6	0	0	1	1	1	0	10.000	0.0	0.0	1	0.0
7	0	0	0	0	0	0	2.000	2.000	0.0	0	0.0
11	0	0	0	0	1	0	10.000	2.000	0.0	1	0.0
12	0	0	0	0	0	0	4.000	4.000	0.0	0	0.0
15	0	0	0	0	1	0	10.000	4.000	0.0	1	0.0
16	0	0	0	0	0	0	6.000	6.000	0.0	0	0.0
18	0	0	0	0	1	0	10.000	6.000	0.0	1	0.0
19	0	0	0	0	0	0	8.000	8.000	0.0	0	0.0
20	0	0	0	0	1	0	10.000	8.000	0.0	0	0.0
21	0	0	0	0	1	0	10.000	10.000	0.0	0	0.0

Computer Printout

(Input data, the initial design and the final design only are reproduced.)

GENERATED NODAL DATA

NODE	BOUNDARY CONDITION CODES						/------NODAL POINT COORDINATES--------/			
NUMBER	X	Y	Z	XX	YY	ZZ	X	Y	Z	T
1	-1	-1	-1	-1	1	-1	0.0	0.0	0.0	0.0
2	-1	-1	-1	-1	0	-1	2.000	0.0	0.0	0.0
3	-1	-1	-1	-1	0	-1	4.000	0.0	0.0	0.0
4	-1	-1	-1	-1	0	-1	6.000	0.0	0.0	0.0
5	-1	-1	-1	-1	0	-1	8.000	0.0	0.0	0.0
6	-1	-1	1	1	1	-1	10.000	0.0	0.0	0.0
7	-1	-1	0	0	0	-1	2.000	2.000	0.0	0.0
8	-1	-1	0	0	0	-1	4.000	2.000	0.0	0.0
9	-1	-1	0	0	0	-1	6.000	2.000	0.0	0.0
10	-1	-1	0	0	0	-1	8.000	2.000	0.0	0.0
11	-1	-1	0	0	1	-1	10.000	2.000	0.0	0.0
12	-1	-1	0	0	0	-1	4.000	4.000	0.0	0.0
13	-1	-1	0	0	0	-1	6.000	4.000	0.0	0.0
14	-1	-1	0	0	0	-1	8.000	4.000	0.0	0.0
15	-1	-1	0	0	1	-1	10.000	4.000	0.0	0.0
16	-1	-1	0	0	0	-1	6.000	6.000	0.0	0.0
17	-1	-1	0	0	0	-1	8.000	6.000	0.0	0.0
18	-1	-1	0	0	1	-1	10.000	6.000	0.0	0.0
19	-1	-1	0	0	0	-1	8.000	8.000	0.0	0.0
20	-1	-1	0	0	1	-1	10.000	8.000	0.0	0.0
21	-1	-1	0	0	1	-1	10.000	10.000	0.0	0.0

EQUATION NUMBERS

N	X	Y	Z	XX	YY	ZZ
1	0	0	0	0	0	0
2	0	0	0	0	1	0
3	0	0	0	0	2	0
4	0	0	0	0	3	0
5	0	0	0	0	4	0
6	0	0	0	0	0	0
7	0	0	5	6	7	0
8	0	0	8	9	10	0
9	0	0	11	12	13	0
10	0	0	14	15	16	0
11	0	0	17	18	0	0
12	0	0	19	20	21	0
13	0	0	22	23	24	0
14	0	0	25	26	27	0
15	0	0	28	29	0	0
16	0	0	30	31	32	0
17	0	0	33	34	35	0
18	0	0	36	37	0	0
19	0	0	38	39	40	0
20	0	0	41	42	0	0
21	0	0	43	44	0	0

THIN PLATE/SHELL ELEMENTS

NUMBER OF ELEMENTS = 15
NUMBER OF MATERIALS = 1
NUMBER OF TEMP CARDS= 1
CONSTRN CODE = 1

MATERIAL PROPERTY TABLE

MATERIAL NUMBER	NUM OF TEMP	SPECIFIC WEIGHT	TEMP	YOUNGS MODULUS	POISSONS'S RATIO	COEFFT OF THERM EXPN	/------------ALLOWABLE STRESSES------------/ TENSION	COMPRESSION	SHEAR
1	1	0.10000	0.0	1.05000E 07	0.300	0.0	12000.00	12000.00	6924.00

ELEMENT LOAD CASE MULTIPLIERS

ELEMENT LOAD CASE NUMBER	PRESSURE	THERMAL EFFECTS	X- ACCELERATION	Y- ACCELERATION	Z- ACCELERATION
1	1.000	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0

THIN PLATE/SHELL ELEMENT DATA

ELEMENT NUMBER	NODE-I	NODE-J	NODE-K	NODE-L	MATERIAL NUMBER	DES VAR NUMBER	NORMAL PRESSURE	REFERENCE TEMPERATURE	DES VAR FRACTION	BETA	BAND WIDTH
1	1	2	7	0	1	1	16.0000	0.0	1.0000	0.0	7
2	2	3	8	7	1	2	16.0000	0.0	1.0000	0.0	10
3	3	4	9	8	1	3	16.0000	0.0	1.0000	0.0	12
4	4	5	10	9	1	4	16.0000	0.0	1.0000	0.0	14
5	5	6	11	10	1	5	16.0000	0.0	1.0000	0.0	15
6	7	8	12	0	1	6	16.0000	0.0	1.0000	0.0	17
7	8	9	13	12	1	7	16.0000	0.0	1.0000	0.0	17
8	9	10	14	13	1	8	16.0000	0.0	1.0000	0.0	17
9	10	11	15	14	1	9	16.0000	0.0	1.0000	0.0	16
10	12	13	16	0	1	10	16.0000	0.0	1.0000	0.0	14
11	13	14	17	16	1	11	16.0000	0.0	1.0000	0.0	14
12	14	15	18	17	1	12	16.0000	0.0	1.0000	0.0	13
13	16	17	19	0	1	13	16.0000	0.0	1.0000	0.0	11
14	17	18	20	19	1	14	16.0000	0.0	1.0000	0.0	10
15	19	20	21	0	1	15	16.0000	0.0	1.0000	0.0	7

B O U N D A R Y   E L E M E N T S

NUMBER OF ELEMENTS = 5

ELEMENT LOAD MULTIPLIERS

A	B	C	D
0.0	0.0	0.0	0.0

BOUNDARY ELEMENT DATA

CONST NUMBER	NODE N	/--NODES DEFINING NI	CONSTRAINT NJ	DIRECTION NK	--/ NL	CODES KD	KR	DISPL D	ROTATION R	STIFF S
1	7	1	0	0	0	0	1	0.0	0.0	2.00D 06
2	12	7	0	0	0	0	1	0.0	0.0	2.00D 06
3	16	12	0	0	0	0	1	0.0	0.0	2.00D 06
4	19	16	0	0	0	0	1	0.0	0.0	2.00D 06
5	21	19	0	0	0	0	1	0.0	0.0	2.00D 06

STRUCTURE LOAD CASE	STRUCTURE LOAD MULTIPLIERS A	B	C	D
1	1.000	0.0	0.0	0.0

NODAL DISPLACEMENT/ROTATION CONSTRAINTS

NODE NO.	LOAD/ CASE	DX	DY	DZ	RX	RY	RZ	-DX	-DY	-DZ	-RX	-RY	-RZ
		------	------	------	------	MAX.ALLOWABLE	DISPLACEMENTS AND ROTATIONS	------	------	------	------	------	------/
21	1	0.0	0.0	0.10000	0.0	0.0	0.0	0.0	0.0	-0.10000	0.0	0.0	0.0

NODAL POINT LOADS

NODE NO.	LOAD CASE	APPLIED LOADS RX	RY	RZ	MX	MY	MZ

TOTAL NUMBER OF EQUATIONS = 44
BANDWIDTH = 17
NUMBER OF EQUATIONS IN A BLOCK = 44
NUMBER OF BLOCKS = 1

****
ANALYSIS OF DESIGN NUMBER 0
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
21	1	0.0	0.0	1.242E-01	1.5011E-04	0.0	0.0
20	1	0.0	0.0	1.153E-01	8.7571E-03	0.0	0.0
19	1	0.0	0.0	1.074E-01	8.1386E-03	-7.8620E-03	0.0
18	1	0.0	0.0	9.057E-02	1.5840E-02	0.0	0.0
17	1	0.0	0.0	8.448E-02	1.4715E-02	-6.0481E-03	0.0
16	1	0.0	0.0	6.703E-02	1.1569E-02	-1.1343E-02	0.0
15	1	0.0	0.0	5.486E-02	1.9487E-02	0.0	0.0
14	1	0.0	0.0	5.125E-02	1.8163E-02	-3.5911E-03	0.0
13	1	0.0	0.0	4.084E-02	1.4342E-02	-6.6958E-03	0.0
12	1	0.0	0.0	2.531E-02	8.7277E-03	-8.5825E-03	0.0
11	1	0.0	0.0	1.823E-02	1.6175E-02	0.0	0.0
10	1	0.0	0.0	1.707E-02	1.5120E-02	-1.0879E-03	0.0
9	1	0.0	0.0	1.369E-02	1.2075E-02	-2.0706E-03	0.0
8	1	0.0	0.0	8.522E-03	7.4622E-03	-2.7074E-03	0.0
7	1	0.0	0.0	2.845E-03	2.4512E-03	-2.3884E-03	0.0
6	1	0.0	0.0	0.0	0.0	0.0	0.0
5	1	0.0	0.0	0.0	0.0	8.1310E-05	0.0
4	1	0.0	0.0	0.0	0.0	1.5218E-04	0.0
3	1	0.0	0.0	0.0	0.0	1.2669E-04	0.0
2	1	0.0	0.0	0.0	0.0	2.1028E-04	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.3000E 00	0.3000E 00	0.3000E 00	0.3000E 00	0.3000E 00	0.3000E 00	0.3000E 00	0.3000E 00	0.3000E 00	0.3000E 00
10	0.3000E 00	0.3000E 00	0.3000E 00	0.3000E 00	0.3000E 00					

ANALYSIS OF PLATE/SHELL ELEMENTS ,CONSTRN CODE = 1

ELEMENT NUMBER	ELEMENT THICKNESS	LOAD COND	/------------MEMBRANE FORCES------------// NXX	NYY	NXY	--------BENDING/TWISTING MOMENTS--------/ MXX	MYY	MXY
1	0.3000E 00	1	0.0	0.0	0.0	-0.2033E 02	-0.2058E 02	-0.1845E 02
2	0.3000E 00	1	0.0	0.0	0.0	-0.2192E 02	-0.6513E 02	-0.2992E 02
3	0.3000E 00	1	0.0	0.0	0.0	-0.3374E 02	-0.1255E 03	-0.2650E 02
4	0.3000E 00	1	0.0	0.0	0.0	-0.4703E 02	-0.1747E 03	-0.1680E 02
5	0.3000E 00	1	0.0	0.0	0.0	-0.5440E 02	-0.2012E 03	-0.5758E 01
6	0.3000E 00	1	0.0	0.0	0.0	-0.4723E 02	0.2016E 02	-0.3735E 02
7	0.3000E 00	1	0.0	0.0	0.0	0.9500E 01	-0.1802E 02	-0.4707E 02
8	0.3000E 00	1	0.0	0.0	0.0	0.1619E 02	-0.2651E 02	-0.3222E 02
9	0.3000E 00	1	0.0	0.0	0.0	0.1799E 02	-0.3214E 02	-0.1119E 02
10	0.3000E 00	1	0.0	0.0	0.0	0.9150E 00	0.7722E 02	-0.2660E 02
11	0.3000E 00	1	0.0	0.0	0.0	0.6663E 02	0.5673E 02	-0.3207E 02
12	0.3000E 00	1	0.0	0.0	0.0	0.7638E 02	0.6481E 02	-0.1138E 02
13	0.3000E 00	1	0.0	0.0	0.0	0.7891E 02	0.1210E 03	-0.9313E 01
14	0.3000E 00	1	0.0	0.0	0.0	0.1169E 03	0.1157E 03	-0.8037E 01
15	0.3000E 00	1	0.0	0.0	0.0	0.1346E 03	0.1428E 03	0.1028E 01

ANALYSIS OF BOUNDARY ELEMENTS - CONSTRAINT FORCES

CONST NUMBER	LOAD CASE	FORCE	MOMENT
1	1	0.0	-0.88759E 02
2	1	0.0	-0.20540E 03
3	1	0.0	-0.31984E 03
4	1	0.0	-0.39125E 03
5	1	0.0	-0.21229E 03

****
EVALUATION OF DESIGN NUMBER 0
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.1001E 01	1	5
MIN	0.4593E 00	1	1

MAX DISP RATIOS	LOAD COND	EQN NUMBER
0.1242E 01	1	43

UNIFORM SCALING OPERATION FOLLOWS

SCALE FACTOR IS 1.075AND DETERMINED BY DISPLACEMENT CONSTRAINTS

DESIGN VARIABLES OF SCALED (CRITICAL) DESIGN ARE

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.3225E 00	0.3225E 00	0.3225E 00	0.3225E 00	0.3225E 00	0.3225E 00	0.3225E 00	0.3225E 00	0.3225E 00	0.3225E 00
10	0.3225E 00	0.3225E 00	0.3225E 00	0.3225E 00	0.3225E 00					

STRUCTURAL WEIGHT= 0.1612E 01

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR DISPT. CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	ACT	-0.33850E-01
2	ACT	-0.17820E 00
3	ACT	-0.50518E 00
4	ACT	-0.94764E 00
5	ACT	-0.12728E 01
6	ACT	-0.22893E 00
7	ACT	-0.27120E 00
8	ACT	-0.23856E 00
9	ACT	-0.18862E 00
10	ACT	-0.27117E 00
11	ACT	-0.27125E 00
12	ACT	-0.25715E 00
13	ACT	-0.57257E 00
14	ACT	-0.95048E 00
15	ACT	-0.22422E 01

NO. OF ACTIVE DISPLACEMENT CONSTRAINTS ARE 1

****
ANALYSIS OF DESIGN NUMBER 9
****

NODAL DISPLACEMENTS AND ROTATIONS

NODE	LOAD	X	Y	Z	XX	YY	ZZ
21	1	0.0	0.0	1.006E-01	1.2931E-04	0.0	0.0
20	1	0.0	0.0	9.714E-02	3.5748E-03	0.0	0.0
19	1	0.0	0.0	9.493E-02	2.6407E-03	-2.4470E-03	0.0
18	1	0.0	0.0	8.008E-02	1.3263E-02	0.0	0.0
17	1	0.0	0.0	8.168E-02	1.1323E-02	1.8846E-03	0.0
16	1	0.0	0.0	8.483E-02	-1.1351E-04	2.1704E-04	0.0
15	1	0.0	0.0	3.748E-02	1.8088E-02	0.0	0.0
14	1	0.0	0.0	3.972E-02	1.9769E-02	1.8241E-03	0.0
13	1	0.0	0.0	7.333E-02	1.4111E-02	2.1848E-02	0.0
12	1	0.0	0.0	1.035E-01	-5.9647E-03	6.0026E-03	0.0
11	1	0.0	0.0	9.176E-03	8.1413E-03	0.0	0.0
10	1	0.0	0.0	9.157E-03	8.5211E-03	-3.3690E-04	0.0
9	1	0.0	0.0	2.416E-02	1.8338E-02	1.1378E-02	0.0
8	1	0.0	0.0	6.331E-02	4.6100E-02	2.1712E-02	0.0
7	1	0.0	0.0	9.141E-02	4.6574E-03	-4.6317E-03	0.0
6	1	0.0	0.0	0.0	0.0	0.0	0.0
5	1	0.0	0.0	0.0	0.0	-7.4002E-04	0.0
4	1	0.0	0.0	0.0	0.0	4.5493E-04	0.0
3	1	0.0	0.0	0.0	0.0	-5.5817E-03	0.0
2	1	0.0	0.0	0.0	0.0	3.4017E-02	0.0
1	1	0.0	0.0	0.0	0.0	0.0	0.0

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.3435E-01	0.3830E-01	0.1282E 00	0.1870E 00	0.5574E 00	0.6805E-01	0.5283E-01	0.9017E-01	0.3666E 00	0.1176E 00
10	0.1245E 00	0.1522E 00	0.2126E 00	0.2788E 00	0.3948E 00					

ANALYSIS OF PLATE/SHELL ELEMENTS ,CONSTRN CODE = 1

ELEMENT NUMBER	ELEMENT THICKNESS	LOAD COND	MEMBRANE FORCES NXX	NYY	NXY	BENDING/TWISTING MOMENTS MXX	MYY	MXY
1	0.3435E-01	1	0.0	0.0	0.0	0.4334E 00	-0.1026E 01	-0.8125E 00
2	0.3830E-01	1	0.0	0.0	0.0	-0.3846E 00	-0.7391E 00	0.1296E 01
3	0.1282E 00	1	0.0	0.0	0.0	-0.1198E 02	-0.3332E 02	0.1827E 02
4	0.1870E 00	1	0.0	0.0	0.0	-0.3294E 02	-0.4828E 02	0.2625E 02
5	0.5574E 00	1	0.0	0.0	0.0	-0.1633E 03	-0.6803E 03	-0.2450E 02
6	0.6805E-01	1	0.0	0.0	0.0	0.3509E 01	0.9749E 01	0.8985E-01
7	0.5283E-01	1	0.0	0.0	0.0	0.7940E 00	0.2054E 01	-0.7270E 00
8	0.9017E-01	1	0.0	0.0	0.0	-0.5965E 01	-0.2916E 01	0.4816E 01
9	0.3666E 00	1	0.0	0.0	0.0	-0.9296E 02	-0.2564E 03	0.2107E 02
10	0.1176E 00	1	0.0	0.0	0.0	0.5670E 01	0.2508E 02	-0.6172E 01
11	0.1245E 00	1	0.0	0.0	0.0	-0.5358E 01	0.7959E 01	-0.1712E 02
12	0.1522E 00	1	0.0	0.0	0.0	0.2313E 00	0.1031E 02	-0.1006E 01
13	0.2126E 00	1	0.0	0.0	0.0	0.1603E 01	0.5855E 02	-0.3622E 01
14	0.2788E 00	1	0.0	0.0	0.0	0.3164E 02	0.9659E 02	-0.1547E 02
15	0.3948E 00	1	0.0	0.0	0.0	0.9637E 02	0.1296E 03	0.6313E 01

ANALYSIS OF BOUNDARY ELEMENTS - CONSTRAINT FORCES

CONST NUMBER	LOAD CASE	FORCE	MOMENT
1	1	0.0	-0.36360E 02
2	1	0.0	-0.53698E 02
3	1	0.0	-0.14642E 03
4	1	0.0	-0.27384E 03
5	1	0.0	-0.18287E 03

****
EVALUATION OF DESIGN NUMBER 9
****

	STRESS RATIO	LOAD COND	DES VARIABLE
MAX	0.1145E 01	1	3
MIN	0.4724E 00	1	12

MAX DISP RATIOS	LOAD COND	EQN NUMBER
0.1006E 01	1	43

UNIFORM SCALING OPERATION FOLLOWS

SCALE FACTOR IS 1.145AND DETERMINED BY STRESS CONSTRAINTS

DESIGN VARIABLES OF SCALED (CRITICAL) DESIGN ARE

VALUES OF DESIGN VARIABLES

	1	2	3	4	5	6	7	8	9	10
0	0.3932E-01	0.4384E-01	0.1468E 00	0.2140E 00	0.6381E 00	0.7789E-01	0.6047E-01	0.1032E 00	0.4197E 00	0.1347E 00
10	0.1425E 00	0.1742E 00	0.2433E 00	0.3191E 00	0.4519E 00					

STRUCTURAL WEIGHT= 0.1094E 01

REDESIGN OPERATION FOLLOWS

OPTIMALITY INDEX OF DESIGN VARIABLES FOR DISPT. CONSTRAINTS

DV NO	ACT/PAS	INDEX
1	PASS	0.12800E-01
2	PASS	0.93611E-02
3	PASS	-0.65865E-02
4	PASS	-0.58290E-01
5	ACT	-0.52696E 00
6	PASS	0.64020E-01
7	ACT	0.60411E-02
8	ACT	-0.19710E-01
9	ACT	-0.56046E 00
10	PASS	-0.16206E 00
11	PASS	-0.27840E 00
12	ACT	-0.45146E 00
13	ACT	-0.37665E 00
14	ACT	-0.43775E 00
15	ACT	-0.44551E 00

NO. OF ACTIVE DISPLACEMENT CONSTRAINTS ARE 1

P.2 Square Plate with Clamped Edges - Problem 2

We consider the same plate as in Sec. P.1, but without any displacement constraints and with an increased allowable normal stress of $\sigma_t^* = \sigma_c^* = 60{,}000$ psi. A change was also made in the finite element mesh: one quarter of the plate was modelled as shown in Fig. P.2.1 (this dispenses with the need for boundary elements.) The problem is now identical to the plate designed in Ref. [12], and provides us with a check of our design procedure.

A comparison of the weight-histories is given in Fig. P.2.2, and the final design variables are shown in Fig. P.2.1. Due to somewhat different details of the two design procedures, the weights of the individual designs are not identical, but the overall convergence characteristics are the same. The distribution of material in Fig. P.2.1 is also similar for the two designs, the differences being mainly due to the larger number of design cycles used in Ref. [12].

The following statement is made on p. 94 of Ref. [12]: "... after fifteen resizings, a discontinuous material distribution has developed which as yet cannot be accounted for." This "discontinuous" distribution is very apparent in the results of DESAP 1 in Fig. P.2.1, where the thicknesses of some elements (underlined figures) have become very small. We disagree, however, with the conclusion that such material distribution cannot be explained; quite to the contrary, the tendency of the thickness to vanish at certain locations is quite sensible.

The very small, underlined element thicknesses in Fig. P.2.1 are an indication that there is a tendency for a "hinge" (line of zero thickness) to develop in the plate. The presence of hinges in the optimal design of statically indeterminate beams is a common occurrence; consequently it may well be that a plate of optimal weight also consists of several "solid" sections connected by hinges.

This problem, as the truss in Sec. L.1, has a very slow convergence rate due to the absence of reasonably large minimum size constraints. In fact, it is unlikely that the fully stressed design will ever be reached unless some constraint is placed on the minimum thickness. The trouble is that the fully stressed design, if it contains hinges, cannot be always reached by a continuous design process without violating the stress constraints at some intermediate stage. As a result, the design variables would eventually oscillate about the fully stressed design.

Upper figures: DESAP 1 after 8 redesigns.

Lower figures: Ref. [12] after 15 redesigns.

Unusually small thicknesses are underlined.

Centerline

Symmetric about the diagonal

							.127
							.102
						.111	.120
						.089	.095
					.077	.095	.106
					.070	.076	.079
				.085	.063	.069	.081
				.038	.058	.057	.046
			.118	.101	.066	.062	.045
			.146	.088	.047	.057	.058
		.023	.060	.016	.047	.151	.090
		.025	.046	.053	.067	.099	.119
	.050	.012	.000	.052	.120	.195	.160
	.020	.020	.024	.055	.106	.148	.177
.000	.003	.042	.044	.071	.171	.246	.218
.019	.027	.035	.050	.082	.134	.192	.232

Clamped (left edge); Centerline (right edge)

Clamped

Figure P.2.1

Final Distribution of Thickness for Plate with Stress Constraints Only.

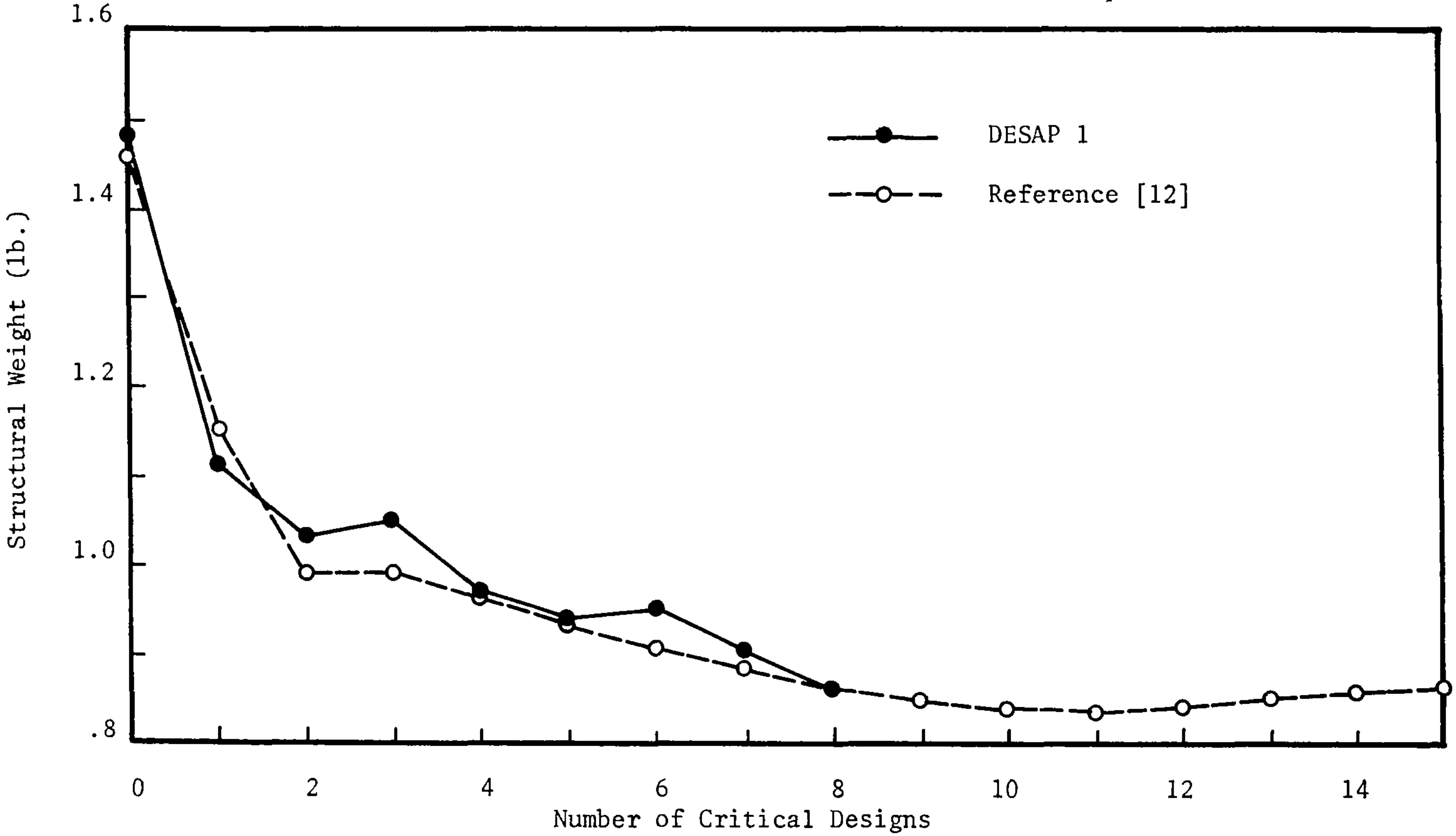

Figure P.2.2

Design History for Plate with Stress Constraints Only.

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